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A MANUAL OF PROBLEMS AND TABLES IN STATISTICS

WITH

NOTES ON STATISTICAL PROCEDURE

BY

FREDERICK C. MILLS

AND

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PREFACE

This *Manual* is designed primarily as a handbook for students of statistics. A number of quite adequate text books on statistics are now available, but for satisfactory instruction in this subject material in addition to that contained in these texts is required. For a course in statistics cannot be given with any degree of success on the traditional lecture, note-book and required-reading basis. In this field one learns by *doing*, and only by doing. If one is to master statistical methods suitable practice in the manipulation of concrete material, in the solution of real problems, is essential. To afford such practice teachers of statistics have felt the need of a collection of problems and statistical materials adapted to the requirements of the present generation of students. In this *Manual* there is presented such a collection. In preparing it, the authors have attempted to gather varied problems and to assemble materials which are interesting in their own right. Only such materials will impart the breath of life to statistical instruction.

While the presentation of such problems has been the primary object of this book, it has seemed advisable to broaden its scope in two directions. In order that students might have between the covers of a single book not only the data of their problems but also the chief tables employed in the solution of these problems, Section VIII has been included. In workable compass there have been presented in this section all the tables required in ordinary statistical analysis. These tables have been included without abbreviation, in order that reference to other sources might not be necessary. In addition to tables available in other compilations there is reprinted here from the *Journal of the American Statistical Association* for June, 1925, the table prepared by Professor Frank A. Ross, giving the sums of the first six powers of the first fifty odd natural numbers, for use in the analysis of time series. We are indebted to Professor Ross for permission to include this table. The corresponding table of the sums of the first six powers of the first fifty natural numbers has been re-computed for the purpose of publication here.

The first section of the *Manual* deals with statistical procedure in certain of its practical aspects. This section presents what is in part a summary of current practices in the planning of an investigation, in the performance of statistical calculations and in the preparation of statistical reports. The suggestions on procedure should prove helpful in the conduct of general statistical studies as well as useful to the student in connection with the problems included in this *Manual*. The sections on

statistical calculations and on the preparation of reports are particularly designed to be consulted by the student in working on his laboratory assignments.

This book has been written as a companion volume to *Statistical Methods* by Frederick C. Mills, and is designed to supplement that book for class room purposes. References to the *Text* relate to that volume. The *Manual* may be used, however, in connection with other statistical texts, for the problems and notes are quite general in character.

Reference is made in Section III to the laboratory sets which may be secured in connection with this *Manual*. The *Text*, the *Manual* and these laboratory sets constitute a unified collection of material written and assembled for a specific purpose. They are designed to meet the needs of teachers and students in elementary and intermediate courses in economic and business statistics. In combination they are an outgrowth of the experience acquired in attempting to develop courses in these fields. It is the hope of the authors that other teachers may find the results of their experience useful.

F. C. M.
D. H. D.

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PROBLEMS AND TABLES IN STATISTICS

I

NOTES ON THE CONDUCT OF A STATISTICAL INVESTIGATION

Every statistical investigation is, to some extent, a law unto itself. The problems which arise are specific, relating to that particular study. Accordingly, it is impossible to lay down a detailed program of action which should be followed unvaryingly in every study. In the matter of general procedure, however, statistical investigations in widely different fields follow much the same course, though problems and technical methods may vary. Familiarity with such a general method of attack is a great aid to the statistical worker, for much waste effort may be avoided by following a fairly systematic procedure. It is the aim of the present section of this *Manual* briefly to outline such a procedure. The steps preceding the actual statistical manipulation of the data are first described. Then follows a discussion of certain practical problems connected with statistical calculations. This is meant merely as a supplement to the detailed account of methods given in statistical text books. A third section is concerned with the content and form of the final report in which the results of an investigation are summarized.

A

Preliminary Phases of a Statistical Investigation

Determination of the Method of Attack. No stage in the entire investigation is more important than the preliminary studies leading to a decision as to the method of attack to be employed in a given case, and a corresponding amount of attention should be given to it. It will be helpful to summarize the steps which may be taken in connection with this decision.

1. *Definition of problem.*

A study of any sort is designed to solve a problem, and if the study is to be direct and effective the problem at issue should be clearly defined in the mind of the investigator. If the problem is hazy and nebulous, methods will lack precision and results will be vague and of doubtful value. The appropriate method of attack cannot be determined until the problem is clearly stated and the various factors and phases of the problem are clearly distinguished. The quickest way to secure this clearness of definition is to set down the actual questions which are to be answered. This process of defining the problem will also help the

investigator to determine whether there is a reasonable basis for assuming that the problem is capable of solution with the means at his disposal.

2. *Study of published data.*

Having defined the problem, it is essential that the investigator determine precisely what has been done in this field and what data are available in published form. Much labor may be saved by this preliminary study, and suggestions may be secured which will aid materially in framing the plans for the investigation or which will lead to modifications in the method and scope of the investigation.

3. *Delimitation of inquiry.*

A final determination of the method of attack cannot be made until the scope of the inquiry has been settled. Having in mind precisely what his problem is, knowing what has been done in the field before, what data are available, and what his resources are, the investigator is in a position to determine just how extensive, or intensive, his study is to be. It is a usual mistake to take in too much territory, to plan too elaborate an investigation at the start, these plans later being modified with loss of time and money. Careful limitation of the inquiry at the start may prevent many of these wastes, and will add to the effectiveness of the investigation.

In deciding upon the scope of the inquiry attention must be given to the question of reliability of results. The accuracy of these results depends upon the number of cases included, but, in general, the accuracy increases in less than direct proportion to the number of observations. Thus to double the accuracy of an arithmetic mean it is necessary to increase the number of cases fourfold. A decision must first be made, therefore, concerning the degree of accuracy required in the results, for the number of cases to be studied and the method of sampling to be employed must be determined upon the basis of this decision. One may find, by attention to this point, that results sufficiently accurate for all practical purposes may be secured with a smaller expenditure of time and money than had been anticipated. (For a discussion of the problem of sampling, see *Text*, Chapter XVI.)

4. *Decision as to method.*

At this stage the general method to be employed may be decided upon, subject, of course, to some later modification as the work proceeds. A number of questions must be answered, for the methods between which choice must be made are diverse and numerous. Certain of these may be suggested:

a. *Is the statistical method appropriate?* Many problems which confront business men and economists are essentially problems of policy, or problems which involve non-quantitive considerations. Statistical

analysis may assist in a solution of such problems or, in some cases, may be quite inappropriate. For example, the question "Have the average earnings of clothing workers in New York City increased as much as the cost of living since 1913?" is capable of direct answer by statistical methods, assuming that the necessary data are available. But the question "Should a tariff be levied on Sicilian lemons imported into the United States?" involves problems of governmental and economic policy, and cannot be answered in purely quantitative terms.

b. Should the study be based on representative data or should it be an intensive inquiry? If the study is to be of the former type, conclusions drawn from the study of a sample (or of a number of samples) will be applied to the entire population from which the sample is drawn. An intensive inquiry, on the other hand, aims to cover the entire population, as the Census of the United States is based upon a house-to-house canvass of the entire country. The usual statistical inquiry is, perforce, conducted on the sampling principle. Some of the precautions to be observed in the conduct of such a study and in the generalization of the results are suggested in the *Text* (pp. 552–3).

c. Shall original data be collected and, if so, by what method? The investigation of the published data may have shown that materials available in governmental or private reports are sufficient for the purposes in mind. On the other hand, these may be inadequate, and it may be necessary to collect original data from the individuals or business units with which the study is concerned. In a comprehensive study such collection of original data is usually done by means of questionnaires filled in by the individuals who are the subjects of the study or by agents who interview these individuals. The character of these questionnaires and the method of employing them are dealt with in more detail below.

In addition to such questions as these, there are other problems of method relating to the technical procedure. Some of these must be settled in advance, after a study of the main problem with which the investigation is concerned, and some can be settled only as they arise in the course of the analysis.

5. *Outline of procedure.*

It is expedient, before the field work or actual compilation of material is commenced, to outline in considerable detail the steps which are to be taken in the course of the investigation. That is, a working plan should be formulated, a plan which should include a statement of any hypotheses which are to be tested. This plan will undoubtedly be modified as the study proceeds but even in tentative form such an outline is requisite to the launching of the enterprise.

All these steps are involved in the determination of a general method of attack, in the laying of the plan of campaign. The next phase of this

first operation is the gathering of material. The data of the problem in question may come from published sources or may be collected in original form for the purposes of the particular study. We shall concern ourselves first with data of the former type.

The Use of Published Data. In using data gathered and published by others, all the limitations attaching to these data must be understood, the conditions under which they were assembled kept in mind, and their precise meaning determined. Certain of the points worthy of attention may be indicated.

1. *Study of sources.*

An important distinction is drawn between *primary* and *secondary* sources of statistical data. A primary source is one publishing data which were gathered, compiled and prepared for publication by the same agent. A secondary source is one in which data are reprinted from a primary source, or are published by an agency other than the one responsible for their original collection. The publications of the U. S. Bureau of the Census are primary sources, while the *Survey of Current Business* published by the U. S. Department of Commerce is an example of a secondary source. Obviously, more reliability attaches to the data derived directly from a primary source, for not only are errors in copying avoided, but the precise meaning of the figures, the conditions under which they were gathered, and the limitations to be borne in mind in interpreting them will be clearly understood by the editors, and are more likely to be explained to the readers. Not only is it important to understand whether the source from which data are secured is primary or secondary, but the general reliability of the agency which gathered the data should be determined. Data may be unreliable because of loose methods of gathering or assembling, or because of conscious or unconscious bias in the responsible agency. The fact of such unreliability should be established, if it exists.

2. *Determination of precise meaning of published figures.*

This involves a study of the source from several different angles.

a. *The definition of the unit.* All quantitative work is based upon the elementary process of *counting*, but to understand the results of a counting operation one must be quite sure of what has been counted. This calls for a precise definition of the unit employed.

One of the most serviceable classifications of statistical units is that given by G. P. Watkins,[1] who divides all statistical units into the following classes and subdivisions:

[1] G. P. Watkins "*Statistical Units*" In: M. T. Copeland, *Business Statistics*, Harvard University Press, Cambridge, 1917.

Classification of statistical units

(1) Individual things

(*a*) Natural kinds

Examples: man, ox, hen

Such natural kinds are much more easily distinguished than artificial units, the meaning of which depends often upon convention. Hence the counting of natural things, such as the number of animals on farms, is likely to be more accurate than a counting of artificial units.

(*b*) Produced kinds; manufactured commodities and instruments

Examples: shoe, door, chair

(2) Units of measurement

(*a*) Units of physical measurement

Examples: ton, gallon, kilowatt hour

Such units are employed as a result of convention. Frequently the same term is employed with varying meanings, which leads to ambiguity and uncertainty in interpreting the results.

(*b*) Pecuniary units

Units of commercial value, such as the dollar, pound and franc, are the least satisfactory of the units with which the statistician must deal, yet these are the most important in ordinary business and economic analysis. The chief defect of this class of unit arises from the changes to which it is subject, as a measure of value, because of changes in the general price level. Index numbers of prices represent an attempt to correct for some of the deficiencies of the pecuniary unit, but such devices fail to remove all the defects of units of this type.

In outlining this classification some of the difficulties connected with the definition of units in statistical analysis have been suggested. In using published data care must be taken that the unit is interpreted precisely as it was by the original investigators. Thus, if one is using Census figures of the number of manufacturing establishments in the United States at a certain date, the precise meaning given to the term "manufacturing establishment" must be understood. Where any ambiguity is likely to exist, the definition given to the enumerators should be published with the data.

b. Determination of degree of error in the data. No compilation can be absolutely accurate, and it should be the duty of any agency gathering and publishing statistical data to indicate the degree of error present in the published figures. Errors may arise from any one of a number of causes, such as faulty collection, difficulty of securing an exact statement from the informant (as when a workingman is asked to state the number of days lost through unemployment during the last twelve months), etc. Serious mistakes may be made if the innocent user of such data is unaware of large errors which may be present.

c. *Other considerations.* An understanding of the significance of published figures calls also for knowledge of the dates to which they relate, of the bases of all percentages employed, in short, for knowledge of all the conditions under which the original study was made. Thus, without knowing whether certain annual figures relate to calendar or fiscal years, comparison of different series is impossible. Tabulated and printed figures always have the appearance of great accuracy, an appearance which is often quite illusory and misleading.

The Gathering of Original Material. If the objects of a given investigation may be attained by the use of published data alone, the investigator is fortunate, for the gathering of data in the first instance is difficult, trying and expensive. On a large scale it usually calls for resources beyond the control of an individual, hence economic and social investigators are largely dependent for their raw material upon the compilations of national and state governments. Difficult as is the initial collection of data, however, it is a method which often has to be resorted to, and when this is the case this stage of the investigation requires the most careful planning.

If the original data are to be gathered by a single individual, on the basis of a personal investigation, special considerations will control the shaping of the study. Usually, however, a wider survey, conducted by means of questionnaires, is necessary, and it is this type of study which calls for brief comment here. It may be said, in passing, that the preliminary planning for a study requiring the extensive use of questionnaires is more important, and must be given correspondingly greater attention, than for any other type of statistical study. Inadequate preliminary planning will always result in heavy wastes, but these losses are likely to be most severe in a study of this type. Previous experience in the framing and circulation of schedules is an almost essential prerequisite to the launching of an enterprise of this sort.

1. *The questionnaire.*

The form of the questionnaire will be determined by the nature of the study, the character of those from whom information is to be obtained, the means adopted for circulating the schedule (i.e., by mail or by personal agents) and, finally, by the nature of the pressure that can be exerted in having the questionnaire filled out. If it is mandatory upon those who receive the schedule to fill it out, as when the investigating agency is a governmental body, more questions, more detailed questions and more inquisitorial questions may be put than when the voluntary coöperation of the informants is sought. In spite of the variations in the form of questionnaires resulting from differences in these respects, certain general principles which are of aid in framing such a schedule have been established by experience.

a. Each question should be simple, brief and specific.

b. Each question should call for an answer of "yes" or "no" or a numerical answer (as the number of children in a family). Exceptions to this rule are sometimes necessary but in all cases the questions should require a clear and unambiguous answer.

c. Inquisitorial questions and those which provoke biased answers should be avoided (e.g. "How many times have you been arrested?").

d. It is advisable to frame questions in such a way that the answers will corroborate each other.

e. The number of questions should be reduced to the absolute minimum consistent with the purposes of the investigation.

These rules will naturally be modified somewhat when the filling out of the questionnaire is compulsory.

If the questionnaire is at all complicated full instructions should accompany it. These may be placed on the schedule, if circulated by mail, or may be prepared in the form of an instruction sheet for enumerators, if agents are employed. This instruction sheet should contain full and exact descriptions of all units employed, as well as information on other possibly ambiguous points. The use of a provisional form in a preliminary investigation may help to indicate the questions which may be misunderstood.

When questionnaires are sent to public agencies or business houses it is sometimes advisable to include duplicate forms that may be retained by the informants.

It is of great importance that the questionnaire be prepared with the tabulation forms in mind. These forms, on which the results secured from the schedules are to be tallied and summarized, are discussed below. The smoothness and expedition of the work of tabulation depend upon a close adaptation of the questionnaires to the tabulation forms, which should be prepared before the questionnaires are finally decided upon.

2. *Editing the questionnaires.*

When the filled out schedules have been returned, and before the process of tabulation is commenced, these schedules should be very carefully edited. By this means incomplete returns may be located and possibly sent back to the informants for completion. Returns which are incorrect, because of obvious inconsistencies or inaccuracies, may be similarly treated. Certain errors, such as faults in addition, may be corrected at once by the editor. An additional purpose in editing should be to secure uniformity in the items which are to enter into a total, or which are to be compared with each other.

The Tabulation of Original Data. Every statistical investigation is undertaken to answer certain questions, in quantitative form. The ultimate answers to these questions may involve extended calculation,

calling for the manipulation of the original data in diverse ways. But before this refined analysis is possible the original data must be transferred from the individual schedules to primary tables, must be organized and, in some degree, condensed in order that the mass of data may be readily available. The character of this preliminary tabulation will be determined by the nature of the data and by the later plans for analysis. Certain general considerations relating to this operation may be briefly discussed. (The treatment below follows, in general, that of Bowley.)

The form of the tabulation sheet, it has been noted, should be decided upon before the schedules are constructed, since the schedule forms must accord with the sheet upon which they are to be summarized. The major objects of the investigation will dictate the form of the tabulation sheet.

1. *General principles.*

   a. *Simple tabulation to secure totals.* Tabulation in its simplest form may involve merely the counting of returns to determine the number of units satisfying a certain condition, as the number of banks with trust departments in a given city, or the number of men unemployed on a certain date. Such simple tabulation would lead to a table of the following type:

TABLE 1

Primary Horsepower used by Manufacturing Establishments in the Middle Atlantic States, 1919

State	Primary horsepower used
New York	2,936,530
New Jersey	1,146,744
Pennsylvania	4,454,386
Total	8,537,660

(From *Abstract of the Census of Manufactures*, 1919.)

Frequently, though totals only are secured, it may be desired to subdivide these aggregates. Thus we might wish to show not only the total amount of power used by manufacturing establishments, but, in addition, to show how much of this power is generated by the user (i.e. is *owned* power) and how much is rented. A slight modification of the above form will enable these subdivisions to be shown, as in the following table:

TABLE 2

Primary Horsepower used by Manufacturing Establishments in the Middle Atlantic States, 1919, Showing Amount of Power Owned and Amount Rented

(1) State	(2) Horsepower owned	(3) Horsepower rented	(4) Total
New York	1,755,986	1,180,544	2,936,530
New Jersey	834,792	311,952	1,146,744
Pennsylvania	3,466,474	987,912	4,454,386
Totals	6,057,252	2,480,408	8,537,660

By a further subdivision, a single table may show a treble classification. Thus we might introduce into the above problem the nature of the power used. This arrangement is illustrated in the accompanying table:

TABLE 3

Primary Horsepower used by Manufacturing Establishments in the Middle Atlantic States, 1919, Showing Owned and Rented Power Classified According to Origin

(1) State	Horsepower owned (2)	(3)	(4)	Horsepower rented (5)	(6)	(7) Total
	Steam engines and turbines	Water wheels and motors	Internal combustion engines	Electric	Other	
N. Y....	1,299,548	343,023	113,415	1,170,977	9,567	2,936,530
N. J....	784,434	12,528	40,830	305,762	6,190	1,146,744
Penn. ..	3,169,790	40,617	256,067	978,621	9,291	4,454,386
Totals	5,250,772	396,168	410,312	2,455,360	25,048	8,537,660

By the addition of two extra columns, sub-totals for "horsepower owned" and "horsepower rented" could have been included, additions which would be desirable in a primary table.

b. The grouping of numerical data in primary tabulation. The usual problem in primary tabulation is that of securing some form of grouping which will render all the data accessible, and will facilitate subsequent calculations. The considerations which enter into the general problem of classification apply to this process. Certain of the central points to be remembered may be summarized here.

(1) Every table, whether primary or derived, should have a definite logical arrangement, should be the expression of some purpose, and should therefore have a definite and clearly marked structure. In deciding upon the arrangement, the general objects of the investigation should be borne in mind, and the arrangement should be significant for these purposes.

(2) If data similar to those to be assembled in this table have been gathered at other times and have been summarized elsewhere, an attempt should be made to secure comparability with these other tables (e.g. by employing similar class-intervals in grouping the data).

(3) The tabulation of the original data in the primary table should involve the minimum loss of detail. Every grouping operation involves some such loss, but by making the class-intervals as small as possible in this preliminary tabulation, this loss may be

reduced. If this is done, greater flexibility is granted to later calculations than would be the case with coarser grouping in the preliminary work.

(4) Clear and unambiguous titles should be given to such primary tables, columns should be numbered and clearly labeled, ample space should be provided for the entries, and totals and subtotals should be distinguished from the other figures.

(5) The use of a "catch-all" class may be necessary in tabulating exceptional entries or ambiguous answers. If numerical data are recorded in such a class (as "Over $10.00"), record of the individual values thus classified should be kept, to facilitate later calculations.

(For a fuller discussion of the problem of classification see *Text*, pp. 64–74.)

2. *Hand tabulation.*

The form of the primary table having been decided upon, the next task is the actual physical tabulation of the individual items. If this task is to be performed by hand, a large tabulation sheet is necessary, arranged in the same form as the primary table but with ample room for the tallying records. The data on the questionnaires are then tallied in the appropriate compartments of this table. If the data are on individual record cards, and the classification is to be on a simple principle which does not call for cross-classification, the sheet for tallying will not be needed. The cards may be immediately separated into classes, by sorting into piles, and the class totals secured in this way. From the tally sheet, or from the classified individual cards, the totals are transferred to the primary table.

3. *Machine tabulation and classification.*

With a limited number of cases the tabulation of data by hand is a simple procedure, but with a fairly large number of cases, to be cross-classified in a number of different ways, it is virtually impossible to proceed by simple hand methods. Census records relating to more than one hundred million individuals cannot be handled in this way, or, if so handled, can be made to yield but a small part of the information they contain. The same difficulty is encountered whenever the number of cases exceeds a few thousand in number. Fortunately, mechanical methods of tabulation and classification have been developed. These methods may be resorted to, at a very moderate unit cost, whenever the data exceed the limited number which can be tabulated by hand. The operations involved in this machine process may be briefly described.

The first operation is the transference of the information carried in each individual schedule to a record card, coded in such a way that a number of diverse facts may be recorded in a very small space. Thus

if information relating to all the banks in New York State had been assembled, the raw material consisting of schedules filled out by the officers of the individual banks, one card would be punched for each bank. This card would be coded so as to carry information as to capital, surplus, loans and deposits, number of depositors, etc.

Once the information has been transferred from the schedules to the cards, the task of analysis is simple. The next step in the machine process is the verification of the punching, so as to eliminate certain impossible combinations. Thus, in a population study, a card might have been punched to show that John Doe, who was "native born of native parents," had a father whose nationality was Irish and a mother whose nationality was English. By a verification process such an error would be located, and the card cast out. Since the cards and the original schedules bear identical numbers, the faulty schedule may be readily found.

The next step is the sorting of the cards by classes, a task for the *sorting machine.* If a classification of banks according to amount of capital is desired, the machine may be set, the cards inserted and run through at the rate of several hundred a minute. Cards of the banks belonging to the different classes are automatically dropped into different slots. This process of sorting may be repeated, to give any sub-classifications needed. The final operation is that of counting and tabulating, an operation performed by a special automatic *tabulating machine.* The actual time necessary for the sorting and counting, after the punching of the original record cards, is negligible.

The service companies handling these machines will undertake a special piece of work in connection with a given investigation, and will advise with those in charge as to the arrangement of the work, the classifications desired, etc. The machines need not be purchased (in fact they are, in general, not sold, but leased) hence no capital outlay for equipment is necessary in employing the method of machine tabulation. A great saving of time and energy is possible, by this method, if any considerable number of cases are to be dealt with.

After the sorting and counting the class totals may be entered in a primary table of the usual type.

B

Statistical Calculations

Statistical work involves, of necessity, a considerable amount of calculation. The manipulation and analysis of the data which have been gathered and assembled by methods discussed in the preceding section constitute a task of some magnitude. In order that this task may be carried through with expedition and accuracy it is essential that due attention be given to the organization of the work of calculation.

This calls for the proper lay-out of the work, in advance of analysis, the preparation of suitable work sheets, and the reduction of all the operations to a smooth, consistent procedure, with the different stages properly interrelated, and with provision made for all possible checks. A slovenly arrangement is fatal to both speed and accuracy. Careful preliminary arrangement will pay for itself many times over in increased accuracy and in saving of time.

The Lay-out of Work; the Work Sheet. The first step in calculation is the lay-out of the data, with reference to subsequent calculations. Before observations are recorded, or transferred from the primary tables, a general scheme should have been prepared, a framework into which the various steps in the later calculations will fit. This scheme, of course, will vary with the data and with the objects of the study, but no matter what the data or the ultimate objects such a scheme is necessary. With the lay-out prepared in advance, the original observations may often be recorded in tabular form immediately adapted to the first stages of the calculation process, thus avoiding the necessity of recopying.

The preparation of suitable work sheets is essential to the organization and carrying through of extensive calculations. The degree of care which may be given to the preparation of such sheets will depend upon the magnitude of the problem and, more particularly, upon whether a series of similar problems is to be attacked. In this latter case, when there will be a fairly constant demand within the organization for the same sort of work sheets, it may be advisable to construct a special model and have special plates made. If this is not expedient, work sheet forms prepared for the market may be found to meet all the requirements of the problem or may be adapted to the purpose in mind. Supplies of those forms which are most generally employed or which have the widest utility should be kept in stock in the statistical laboratory. A third method of securing the needed forms is the simple and convenient one of ruling standard sheets to conform to the desired model.

In organizing a work sheet attention should be given to the proper spacing of columns and lines and to the clear and unambiguous heading of all columns, so that there shall be no uncertainty as to the derivation and meaning of the data or calculations recorded therein. All columns should be numbered to permit of ready reference. It is often possible to insert work sheets directly into an adding machine, thus having the printed record on the sheet. This may greatly facilitate checking and later calculations. The size, form and spacing of the work sheet should be adapted to this purpose, if the adding machine record is to be utilized. Forms appropriate to the computation of the primary statistical measures are exemplified in the *Text*.

Methods and Accuracy of Calculations. The procedure, with respect to calculation, will have been decided upon in planning the lay-out of work and the work sheets. The general method, in practically all cases involving the handling of a considerable mass of data, will call for the tabular arrangement of original data and of all subsequent calculations. The tabular arrangement is far better adapted to a consistent procedure than any less formal method, and in handling masses of material such a procedure is necessary. Once such a scheme has been prepared, the carrying out of the calculations is a fairly simple matter. In the original lay-out of such a scheme all possible methods for reducing labor should be resorted to. It would call for far more extended treatment than is possible at this point to discuss in detail, or even to enumerate, all such labor-saving methods, but certain general aids to calculation should be listed.

1. *Aids to calculation.*

The standard tables which may be employed to facilitate numerical calculations are familiar to all students, but often not sufficiently familiar so that they are used readily and accurately. *Tables of logarithms* are, of course, indispensable. With mechanical calculators generally available logarithms are not widely employed for the operations of multiplication and division, but they still offer the simplest method of raising to powers and extracting roots, except where prepared tables of powers and roots are available. Logarithms will generally be employed in the calculation of the geometric mean of a frequency series (see *Text*, p. 136 for example). In fitting curves in the equations to which the x or y variable appears in logarithmic form such tables are necessary (see *Text*, p. 291 for example). For graphic presentation, the student should remember, the use of logarithmic paper will often render unnecessary the use of logarithms. A table of five place logarithms is included in this *Manual.*

Tables of squares, square roots, and reciprocals are of equally wide utility. The most complete set of tables of this type is that bearing the name of Barlow (Barlow's *Tables of Squares, Square Roots, Cubes, Cube Roots and Reciprocals*), covering numbers up to 10,000. The uses of such tables in statistical work are many, and need no detailed description. Attention may be called to one use of the tables of reciprocals. When a problem calls for dividing a series of numbers by a constant base (as in computing percentages), the reciprocal of the constant base may be employed, and the operation of division supplanted by that of multiplication (i.e., $6 \div 3$ is equivalent to $6 \times \frac{1}{3}$). By placing this reciprocal as the multiplier on any of the mechanical calculators now on the market, the required percentages may be run off in very short order. A table of squares, square roots and reciprocals of the numbers 1 to 1,000 is contained in this *Manual.*

The *slide rule* is a useful device for multiplying, dividing, raising to powers and extracting roots. Mechanical calculators have in part superseded slide rules in recent years, but as ready and always accessible aids the latter are still extremely useful.

Crelle's Tables facilitate multiplication and division, when mechanical calculators are not available.

Most important today are the various *calculating machines* now put on the market at prices which make them available to all who have extensive calculations to perform. By the use of hand-driven or electric machines the labor of calculation which accompanies all quantitative work has been immeasurably lessened. Statistical methods are being adapted to these machines, and more will be done in this direction. An example of how methods may be so modified is presented below.

Use of the adding machine in the computation of the mean deviation. A simple formula to employ in securing the mean deviation of ungrouped data, using an adding machine, is given by Kelley.[1] When the mean deviation from the mean is desired this formula is

$$M.D. = \frac{2(FM - \Sigma m_L)}{N}$$

where M represents the arithmetic mean, F the number of measures lying below the mean, m an individual item, Σm_L the sum of the items lying below the mean and N the total number of cases. In the usual employment of this formula, working with data not arranged according to magnitude, the individual items would first be listed on the adding machine, M being computed from the total. Then all items less than the mean are added, the number of such items and their total value being determined. From the above formula the mean deviation from the mean may be determined at once.

The following example will illustrate the method. Given the items

6, 8, 4, 12, 9, 5, 15

Adding these items as they stand we secure 59 as the total and $8\frac{3}{7}$ as the mean. Adding now the four items which fall below this value, we secure 23 as the total. Substituting these values in the formula, we have

$$M.D. = \frac{2\,(4 \times 8\frac{3}{7} - 23)}{7}$$

$$= 3\frac{3}{49}$$

When the items are arranged in order of magnitude but one listing will serve, several sub-totals being taken in the neighborhood of the mean, as approximately located by inspection. Thus for the items given above we would have, as the adding machine record,

[1] Truman L. Kelley, *Statistical Method* (N. Y. Macmillan, 1923), 70–75.

4
5
6
15s
8
9
32s
12
15
59*

The mean is computed, and found to be $8\frac{3}{7}$. Using the nearest subtotal we readily determine the sum and the number of the items falling below the mean in value. It is apparent that there are four such items and that their total is 23. Substituting in the formula, as given above, the mean deviation is computed.

This formula may be readily derived by representing the individual deviations from the mean by $M - m_1$, $M - m_2$, etc., for those items which fall below the mean, and by $m_5 - M$, $m_6 - M$, etc., for those which exceed the mean in value. Thus we have, for the above seven items,

$$M.D. = \frac{(M - m_1) + (M - m_2) + (M - m_3) + (M - m_4) + (m_5 - M) + (m_6 - M) + (m_7 - M)}{N}$$

This may be simplified to the formula as given. A modification of this formula, to permit a similar calculation of the mean deviation from the median, is given by Kelley, in the reference cited.

The usual formulas for the measures of correlation lend themselves readily to machine calculation. The calculations to be performed, following either the least squares method or the product-moment method as outlined in the *Text* (see pp. 401–404 for summary), may be carried through rapidly on ordinary calculating and adding machines. Several special machines for use in calculating measures of correlation have recently been perfected.[1] In computing measures of multiple correlation use may be made, in addition, of tabulating and sorting machines. If a large number of cases are to be handled the use of such machines is imperative. The method will be found explained in detail in a publication of the United States Department of Agriculture.[2]

2. *The checking of numerical calculations.*

In the organization of statistical work full provision must be made for the checking and cross-checking of all calculations. The work of no mortal person is free from error, and the inevitable errors in any extensive series of calculations may be avoided, or reduced to a minimum, only by the careful checking of all operations. By recognizing in advance

[1] *Journal of the American Statistical Association*, Dec. 1925.

[2] *The Use of Punched Card Tabulating Equipment in Multiple Correlation Problems*, Bradford B. Smith. (Prepared for the use of statisticians of the Bureau of Agricultural Economics, U. S. Department of Agriculture.)

the necessity of such checking, methods may be adopted which will enable checks to be most effectively applied.

Two types of checks are available to the quantitative worker. Calculations may be checked, first, by a repetition of the operations. If this is done, it is advisable that the second operation be performed by a person other than the original calculator, or, if that is not possible, that the sequence of operations be altered when the check is made, or that a slightly different method of securing the same result be employed. Thus a column may be added in the opposite direction from that first followed, or multiplier and multiplicand may be reversed. The second type of check is that which provides a numerical test of the accuracy of given calculations. That is, certain values required merely for checking purposes may be computed, in addition to those actually required in the given problem. The Charlier check upon the operation of computing the standard deviation (see *Text* p. 156) is an example of this type. A more elaborate example, in which a whole series of checks is provided for testing the accuracy of the work at various stages, is afforded by the Doolittle method of solving simultaneous equations (see *Text* p. 578). Checks of this latter type should be employed whenever available.

Perhaps more important than all such checks is the habit, on the part of the operator, of mentally verifying the major results of his calculations as he proceeds. If two figures are to be multiplied the operator should determine, by inspection, the approximate value of the product and the number of decimal places it should contain. In any arithmetic operation the same rough check should be employed, for by this means the most serious errors, such as arise from the misplacing of decimal points, may be prevented. Many checks of the same sort are possible in connection with statistical calculations. Thus the standard deviation may be compared with the range (the latter will not, in general, be more than six times the standard deviation), and geometric, harmonic and arithmetic measures may be checked against each other, if all have been computed in a given instance. Inconsistencies in the results usually reveal the most serious errors, and careful watch should be kept for such discrepancies.

By plotting the results of calculations errors may often be detected. If a serious mistake has been made in fitting a line to certain data, it will be immediately evident when data and line are plotted. If the ordinates of a fitted curve have not been correctly determined, breaks in the smoothness of the curve will usually reveal the errors when the curve is plotted.

In seeking to avoid mistakes no one precept is more important than this: *Keep a neat, careful and complete record of all calculations.* This is not only necessary as an aid to subsequent checking but it is essential to accurate calculation. When a series of computations is laid out in proper form and performed in a systematic fashion, the probability of

error is very much less than when the computations are performed in a slipshod, unsystematic fashion.

3. *The accuracy of measurements and calculations.*

A question that arises in connection with every piece of calculation is that of the degree of refinement to which the results should be carried, or the degree of accuracy which should be sought. Failure to take account of this problem usually leads to a waste of time in carrying the calculations out to an unnecessary degree, and to the securing of results which have a fictitious appearance of accuracy. The first consideration, in approaching this problem, relates to the accuracy of the original observations.

The Accuracy of Measurements. The operation of measurement involves in all cases a comparison of magnitudes. Thus a given magnitude, the height of John Smith, is compared with certain standard units of linear measurement, the foot and the inch. In setting up such a comparison absolute accuracy is never possible. We may say that John Smith is 5 feet 8 inches tall, which means that his height lies between 5 feet 7.5 inches, and 5 feet 8.5 inches. The *absolute error* (the difference between the observed and the true values) may in this case be as great as .5 inches. Or, employing more accurate instruments, we may report that John Smith's height is 5 feet 8.3 inches. This means that his height is between 5 feet 8.25 inches and 5 feet 8.35 inches. The absolute error in this case may be as great as .05 inches.

In interpreting recorded measurements, therefore, due attention must be paid to the number of *significant figures*, that is, figures which are known to be correct. There are certain standard rules which should be followed in recording and interpreting measurements with respect to the significant figures. Only the number of correct figures should be recorded, with zeros added, of course, to indicate the absolute magnitude of the measurement. Thus if a distance is recorded as being 4300 feet, it means that the true distance which was measured lies between 4250 and 4350 feet. There are only two significant figures in this example. If wheat production in the United States in a certain year is given as 856,000,000 bushels the amount is recorded to three significant figures. Similarly, if a magnitude is given as .0472, there are but three significant figures, the zero being added, as in the above examples, to indicate the absolute magnitude of the measure. A zero added to the right of the last recorded figure, however, if to the right of the decimal point, is significant, in indicating the degree of accuracy. Thus the value 12.50 has four significant figures, the last zero being added to show that the true value of the recorded magnitude is between 12.495 and 12.505. If it had been given as 12.5, this would be interpreted to mean that the true value lies between 12.45 and 12.55.

Determining the Accuracy of Computations. When observations are combined, it is important to be able to determine the degree of accuracy

of the resultant figures. This may be determined approximately if the accuracy of the original observations is known. The problem may be considered with respect to the four chief arithmetical operations.

Addition. In the addition of measurements, no attempt should be made to give the total an appearance of greater accuracy than the constituent items. If these items differ in accuracy, the total is no more accurate than the least accurate measurement. Thus, in the addition of the following four figures:

$$\begin{array}{r} 25.23 \\ 1610.1 \\ 17.375 \\ 2. \\ \hline 1654.705 \end{array}$$

the total should be rounded off to 1655. It would give a quite spurious impression of accuracy to present the sum as 1654.705.

The actual limits within which the true sum falls may be readily determined by computing the maximum sum and the minimum sum which could be secured from the observations in question. Thus, substituting for each of the above values the maximum value which the quantity in question might have, we secure

$$\begin{array}{r} 25.235 \\ 1610.15 \\ 17.3755 \\ 2.5 \\ \hline 1655.2605 \end{array}$$

Substituting the minimum values, we have

$$\begin{array}{r} 25.225 \\ 1610.05 \\ 17.3745 \\ 1.5 \\ \hline 1654.1495 \end{array}$$

To have presented the original total as accurate to the third decimal place would have been clearly faulty. Nor would it have been accurate to have rounded off the individual items before adding, until their accuracy was equal to that of the least accurate item. The rounding off should be done after the total is secured, as the fullest possible use is thus made of the knowledge we have.

If the limits of error of the individual items (i.e., the differences between the maximum and minimum possible values) be added, it will be found to total 1.111, equal to the difference between the maximum

and minimum possible values of the sum of the items. *The error of a sum may be determined by adding the errors of the constituent items.* (The range between the maximum and minimum possible values is obviously twice the maximum *absolute error*, as defined above.)

Subtraction. By precisely analogous reasoning it may be shown that the limits of error of the difference between measurements may also be determined by adding the limits of error of the individual items. (If the absolute errors of the original measures, with their signs, are known, the absolute error of the difference may be found by taking the *algebraic difference* between the two absolute errors.)

Multiplication. If it is desired to know precisely the accuracy of the product secured by multiplying one quantity by another, it is possible to employ the process illustrated above, namely, to determine the maximum possible value and the minimum possible value. Thus as the maximum possible value of the product of 11.30 and 2.3 we have 11.305×2.35, or 26.56675. As the minimum possible value of the product we have 11.295×2.25, or 25.41375. The product of the numbers as given, 11.30×2.3 is 25.990. Comparing this with the two limits as computed above, we have 26 as the product expressed in terms of significant figures only.

The approximate error of a product may be determined readily by adding the *relative errors* of the two factors. (The *relative error* is the ratio of the absolute error to the given value of a measure.) Thus for the above multiplicand the relative error is $\frac{.005}{11.30}$, or .044 per cent; for the multiplier it is $\frac{.05}{2.3}$, or 2.174 per cent. The relative error of the product is .044 plus 2.174, or 2.218 per cent; 2.218 per cent of 25.990 is .57646, the absolute error of the product. (This value, it will be noted, is just half the difference between the maximum and minimum possible values of the product, as computed directly.) When the relative errors of the two factors differ materially, the relative error of the product may be taken as equal to the relative error of the least accurate figure. Thus in the above example it would be permissible to take as the relative error of the product the value 2.174 per cent, the error of the multiplier 2.3.

Division. By similar reasoning to the above it may be shown that the approximate error of a quotient may be found by taking the sum of the relative errors of dividend and divisor, without regard to sign, multiplying by two, and finding the corresponding absolute value. (If the signs of the relative errors are known, the relative error of the quotient may be found by taking the *algebraic difference* between the relative errors of divisor and dividend.) When the relative errors of dividend and divisor differ materially, the relative error of the quotient may be taken as equal to the relative error of the least accurate of these measures.

In the physical sciences and in engineering fairly standard practices have been established in the matter of recording results, so that the user of published figures may know what the reliability of a given measure is. In the exact physical sciences it is customary to present numerical values with one more figure than those which are known to be significant. The next to the last figure, that is, may be taken to be correct. In recording engineering calculations, on the other hand, only the significant figures are given. The last figure may be taken to be correct, within half a unit, as in the examples given above. No standard practice has been established in statistics, but it would seem expedient in general to follow the engineering practice, recording only those figures which are known to be significant, the last one not being in error by more than half a unit. In the actual calculations, however, one additional figure should be retained, this being dropped when the final result is recorded.

When a statistical measure such as the mean, the standard deviation or the coefficient of correlation has been derived, a useful rule laid down by T. L. Kelley [1] may be followed. The rule is *keep to the place indicated by the first figure of one-half the probable error.* Thus if the arithmetic mean of a given distribution is calculated to be 36.5321, with a probable error of .642, the recorded value of the mean should be 36.5. For one-half the probable error is .321, the first figure being in the column of tenths. With this probable error it is useless to carry the value of the mean beyond the first decimal place. In all calculations the value of the mean would be carried to one additional place, but this would be dropped in recording.

The Use of Tables in Calculations. Reference has been made above to two classes of tables which are employed constantly in statistical work, tables of logarithms, and tables of squares, square roots and reciprocals. In addition to such general purpose tables, a number of special tables, for aiding particular operations, have been prepared. These should be familiar to every statistician, and should be readily accessible in statistical laboratories, for by their aid much laborious calculation may be avoided. It would be impossible, within the limits of the present discussion, to include a complete account of the tables of this type which are in existence, but a brief summary may indicate the nature of the help which is afforded by such material. The use of these tables may be discussed in connection with specific statistical calculations, these being presented in the order of their treatment in the *Text.*

For the relatively simple operations involved in the computation of averages, measures of dispersion and skewness, and index numbers, no tables beyond tables of squares and tables of logarithms are ordinarily needed.

[1] "How Many Figures are Significant." *Science,* Dec. 5, 1924, p. 524.

(1) *Tables to employ in the analysis of time series.*

In fitting lines of trend to time series it is necessary to secure the powers of certain numbers, and the sums of these powers. Barlow's *Tables* are available for securing the squares and the cubes of natural numbers. Table XXVII of Pearson's *Tables for Statisticians and Biometricians* gives the second to the seventh powers of the natural numbers from 1 to 100. Table XXVIII of Pearson's *Tables* gives the sums of the powers from one to seven of the first hundred natural numbers. This table is particularly useful in securing the sums of the powers of x, when x represents time in connection with the fitting of a line of trend. Table C in Section VIII of this *Manual* gives the second to the sixth powers and Table D gives the sums of the first six powers of the first fifty natural numbers.

It is possible to secure the sums of the various powers by formulas when tables are not readily available.[1] We may denote by t the total number of terms in the series 1, 2, 3, 4, 5, 6 . . ., and by S_1, S_2, S_3, S_4, S_5 and S_6 the sums of the first, second, third, fourth, fifth and sixth powers of these numbers. The required formulas [2] are

$$S_1 = \frac{t(t+1)}{2}$$

$$S_2 = \frac{2t^3 + 3t^2 + 1}{6} = S_1\left(\frac{2t+1}{3}\right)$$

$$S_3 = \frac{t^4 + 2t^3 + t^2}{4} = (S_1)^2$$

$$S_4 = \frac{6t^5 + 15t^4 + 10t^3 - t}{30} = S_2\left(\frac{3t^2 + 3t - 1}{5}\right)$$

$$S_5 = \frac{2t^6 + 6t^5 + 5t^4 - t^2}{12} = (S_1)^2\left(\frac{2t^2 + 2t - 1}{3}\right)$$

$$S_6 = \frac{6t^7 + 21t^6 + 21t^5 - 7t^3 + t}{42} = S_2\left(\frac{3t^4 + 6t^3 - 3t + 1}{7}\right)$$

When a line of trend is to be fitted to a time series, with n observations, n being odd, and if the origin be taken mid-way in the time series, then t, of the above formulas, is equal to $\frac{n-1}{2}$. (Thus if there are data for eleven years, the origin will fall at the sixth year and there will be five observations on each side of the origin. In this case n will equal 11 and t will equal 5.) Professor Ross has adapted the above formulas to

[1] An admirable summary of such formulas has been prepared by Frank A. Ross, "Formulae for Facilitating Computations in Time Series Analysis." *Journal of the American Statistical Association*, March, 1925. The formulas in the present and immediately succeeding sections are taken from this summary.

[2] Several of these formulas are given on p. 279 of the *Text*. The symbol n as used in the *Text* corresponds to the symbol t as used above, and does not have the same meaning as n in the formulas given on pp. 24 and 25 of this *Manual*.

this case, so that the value of n may be inserted directly. The revised formulas for the sums of the powers of x (deviations from the mean being represented by x) are

$$\Sigma x = 0$$
$$\Sigma x^2 = \frac{n^3 - n}{12}$$
$$\Sigma x^3 = 0$$
$$\Sigma x^4 = (\Sigma x^2)\left(\frac{3n^2 - 7}{20}\right)$$
$$\Sigma x^5 = 0$$
$$\Sigma x^6 = (\Sigma x^2)\left(\frac{3n^4 - 18n^2 + 31}{112}\right)$$

where x is one time unit.

When the number of years in a time series is even it is convenient, in taking the origin at the middle of the period, to employ a time unit of one-half year (See *Text* p. 279). When this is done the time series, beginning at the origin, will be of the type 1, 3, 5, 7, 9 . . ., and the formulas given above and the tables referred to will not be directly applicable in getting the sums of the powers of x. A table giving the summation of powers of the odd natural numbers has been prepared by Professor Ross, and part of it is reprinted with his permission as Table E in Section VIII of this *Manual*.[1] These sums may also be computed from formulas, which are given below. In these formulas t denotes the number of terms in the series 1, 3, 5, 7, . . . $(2t - 1)$, while ${}_oS_1$, ${}_oS_2$, ${}_oS_3$, ${}_oS_4$, ${}_oS_5$, ${}_oS_6$ represent the sums of the first, second, third, fourth, fifth and sixth powers of these numbers.

$${}_oS_1 = t^2$$
$${}_oS_2 = \frac{4t^3 - t}{3}$$
$${}_oS_3 = 2t^4 - t^2 = {}_oS_1\,(2t^2 - 1)$$
$${}_oS_4 = \frac{48t^5 - 40t^3 + 7t}{15} = {}_oS_2\left(\frac{12t^2 - 7}{5}\right)$$
$${}_oS_5 = \frac{16t^6 - 20t^4 + 7t^2}{3} = {}_oS_1\left(\frac{16t^4 - 20t^2 + 7}{3}\right)$$
$${}_oS_6 = \frac{192t^7 - 336t^5 + 196t^3 - 31t}{21} = {}_oS_2\left(\frac{48t^4 - 72t^2 + 31}{7}\right)$$

When the number of observations, n, in a time series is even, and the origin is taken mid-way in the time series, $t = \frac{n}{2}$. Representing by x deviations from the mean, the x unit being one-half the time unit, Professor Ross gives

[1] This table first appeared in the *Journal of the American Statistical Association*, March, 1925.

$$\Sigma x = 0$$

$$\Sigma x^2 = n\left(\frac{(n+1)(n-1)}{3}\right)$$

$$\Sigma x^3 = 0$$

$$\Sigma x^4 = (\Sigma x^2)\left(\frac{3n^2-7}{5}\right)$$

$$\Sigma x^5 = 0$$

$$\Sigma x^6 = (\Sigma x^4)\left(\frac{3n^4-18n^2+31}{7}\right)$$

In fitting certain types of curves it is necessary to compute the sums of the logarithms of x, and the sums of the squares of the logarithms of x. Appendix V of Pearl's *Medical Biometry and Statistics* contains a useful table which gives the sums of the first and second powers of $\log x$ and the sums of $x \log x$ for the natural numbers from 1 to 100.

A curve of the ordinary exponential type, $y = ab^x$, may be fitted by reducing the equation to logarithmic form. If the fitting be by least squares, this means the securing of a curve from which the sum of the squares of the logarithmic deviations is a minimum. Professor James W. Glover has employed another method of fitting a curve of this type, and has prepared a table which greatly simplifies the task of determining the constants in the equation to the curve of best fit. This table is found on pages 468–481 of Glover's *Tables of Applied Mathematics.*

(2) *Tables to employ in the measurement of correlation.*

Extensive use is found for prepared tables in this field only in the calculation of coefficients of partial correlation. (The method is explained in the *Text*, pp. 507 *et seq.*) In computing such coefficients by one method it is necessary to calculate values of $1 - r^2$ and $\sqrt{1-r^2}$. Values of $1 - r^2$, for values of r varying from .001 to .999 are contained in Table VIII of Pearson's *Tables.* Somewhat more extensive are J. R. Miner's *Tables of* $\sqrt{1-r^2}$ *and* $1 - r^2$ *for Use in Partial Correlation and in Trigonometry* (Johns Hopkins Press), which are almost indispensable for work in this field. The values of $\sqrt{1-r^2}$, it has been pointed out by Holbrook Working, may be secured from ordinary tables of the trigonometric functions. If $r = \cos a$, $\sqrt{1-r^2} = \sin a$.

(3) *Tables to employ in the analysis of frequency distributions and the fitting of frequency curves.*

Numerous tables are available for use in connection with the study of frequency distributions, but of these only those most commonly employed will be mentioned here. For more detailed treatment of this subject, readers are referred to the valuable *Introduction* to Pearson's *Tables for Statisticians and Biometricians.*

The characteristics of the "normal" or Gaussian curve have been studied in great detail. W. F. Sheppard first expressed the normal probability integral in terms of the standard deviation, and his tables, first published in *Biometrika*, are of basic importance in statistical work. Tables II and III of Pearson's *Tables* are Sheppard's work. In Table II the areas and ordinates of the normal curve are expressed in terms of abscissa, while Table III gives abscissa and ordinate in terms of differences of areas. Supplementing Sheppard's tables is the Kelley-Wood Table of the Normal Probability Integral, given as Appendix C to *Statistical Method*, by Truman L. Kelley. In the Kelley-Wood Table abscissa and ordinate are given in terms of areas. Glover's book, *Tables of Applied Mathematics*, contains tables of the areas, ordinates and second to eighth derivatives of the normal curve of error. Table F in Section VIII of this *Manual* gives areas and ordinates in terms of abscissa.

A few of the many applications of these tables are illustrated in the *Text* (Chapter XV). Therein it is shown how theoretical frequencies may be computed either from a table of ordinates or from a table of areas. The table of ordinates employed in the *Text*, however, is of a different type from that appended to this *Manual*, and it seems advisable to illustrate at this point the use of the latter table.

The formula for the normal curve of error, as usually given, is

$$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

Dividing through by N and multiplying by σ, we have

$$\frac{y\sigma}{N} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

We may use the symbol z for $\frac{y\sigma}{N}$. This value is that which is recorded in Sheppard's Tables, and which is given in Table F of this *Manual*. Deviations from the mean are recorded in units of the standard deviation in this table (i.e. abscissal values are those of $\frac{x}{\sigma}$), and corresponding values of z are given as the tabled entries.

Since

$$z = \frac{y\sigma}{N}$$

then

$$y = z \times \frac{N}{\sigma}$$

Thus, from values of z and $\frac{N}{\sigma}$ the theoretical frequencies in a given case may be readily determined. The procedure is illustrated in Table 4, employing the data utilized in Table 126 of the *Text*.

TABLE 4

Computation of Ordinates of Normal Curve Fitted to Frequency Distribution of 995 Telephone Subscribers Classified According to Annual Message Use

(1) Number of messages	(2) Actual frequency	(3) Deviation from M (in class intervals)	(4)	(5)	(6) Computed frequency
m	f	x	$\frac{x}{\sigma}$	z	$\frac{N}{\sigma} \times z$
25	0	− 9.0392	− 3.06	.0036951	1.25
75	1	− 8.0392	− 2.72	.0098712	3.33
125	9	− 7.0392	− 2.38	.0234910	7.92
175	19	− 6.0392	− 2.05	.0487920	16.44
225	38	− 5.0392	− 1.71	.0924591	31.15
275	50	− 4.0392	− 1.37	.1560797	52.59
325	95	− 3.0392	− 1.03	.2347138	79.09
375	85	− 2.0392	− .69	.3144317	105.95
425	115	− 1.0392	− .35	.3752403	126.44
475	132	− .0392	− .01	.3989223	134.41
525	144	.9608	.32	.3790305	127.71
575	116	1.9608	.66	.3208638	108.11
625	79	2.9608	1.00	.2419707	81.53
675	54	3.9608	1.34	.1625551	54.77
725	31	4.9608	1.68	.0972823	32.78
775	11	5.9608	2.02	.0518636	17.48
825	5	6.9608	2.36	.0246313	8.30
875	6	7.9608	2.70	.0104209	3.51
925	2	8.9608	3.03	.0040486	1.36
975	1	9.9608	3.37	.0013639	.46
1025	1	10.9608	3.71	.0004093	.14
1075	1	11.9608	4.05	.0001094	.04
	995				994.76

The standard deviation of this distribution, in class-interval units, is 2.953. The arithmetic mean is 476.96. Dividing by the standard deviation the actual deviations of the various class mid-points from the mean, we secure the values given in column (4). Referring to Table F, the corresponding values of z are determined. For $\frac{N}{\sigma}$ we have $\frac{995}{2.953}$ or 336.94548. Multiplying the values of z, as determined for the various classes, by this quantity, the computed frequencies given in column (6) of Table 4 above are secured. (For certain classes the frequencies as given above differ slightly from those given in Table 126 of the *Text*. The above frequencies are calculated with a slightly higher degree of accuracy.)

An illustration of the use of the table of areas is given in the *Text* (pp. 537–540) and need not be repeated here.

Prepared tables are necessary in applying the Chi-square test for goodness of fit. The necessary values are given in Table XII of Pear-

son's *Tables*. An illustration of the application of this test is given in the *Text* (pp. 543–545).

In determining the type of frequency curve (Pearsonian) to fit to a given distribution, certain criteria are employed. These are conveniently summarized on page lxiii of Pearson's *Tables*, while a diagram to assist in the determination of the appropriate curve type is given on page 66 of that work. Elderton's *Frequency Curves and Correlation* and Kelley's *Statistical Method* also discuss the criteria of curve type and contain examples of the fitting process. (See *Text* p. 532 for an example of the application of such criteria.)

In computing the moments of a frequency distribution advantage may be taken of a short method devised by G. F. Hardy. This method is explained by W. P. Elderton in *Frequency Curves and Correlation* (pp. 19–23). A check upon the calculations involved in computing the moments of a frequency distribution is afforded by Table L of Pearson's *Tables*, giving the fourth moments of various group-frequencies.

In fitting generalized frequency curves of the types developed by Karl Pearson use will be found for many of Pearson's tables not mentioned here. In addition, *Tables of the Complete and Incomplete Gamma Function* are essential for advanced work. Titles of such tables are given in the list of references.

A great many tables and diagrams for use in determining the probable errors (or standard errors) of the chief statistical measures will be found in Pearson's *Tables*. These measures of reliability may all be computed from formulas, but much labor may be saved by the use of prepared tables. The following tables in Pearson may be mentioned:

Table V. Probable errors of means and standard deviations.

VI. Probable errors of coefficients of variation.

VIII. Probable error of a coefficient of correlation: Table to facilitate the calculation of $1 - r^2$.

XXXVII and XXXVIII. Probable errors of frequency constants, β_1 and β_2.

XL. Probable error of frequency constants: Probable error of the distance between mean and mode for given values of β_1 and β_2.

XLI. Probable errors of frequency constants: Probable error of the skewness for given values of β_1 and β_2.

XLIII. Probable errors of the frequency constants: Probable error of the criterion of type, κ_2, for given values of β_1 and β_2.

The Use of Tables: Elementary Principles of Interpolation. All tables are of necessity limited to a certain restricted number of values of the functions recorded. Thus, reading from the table of logarithms appended (Table A), we have

Argument Natural number	*Function* Logarithm
22.82	1.35832
22.83	1.35851
22.84	1.35870
22.85	1.35889
22.86	1.35908

If it is desired to secure the logarithm of a number between those given above, it is necessary to *interpolate* between the intervals of the argument. That is, one must find that value of the function corresponding to the particular value of the argument and consistent with the tabled values of function and argument. This problem arises in using every table, and a similar problem is faced in many other forms in statistical analysis. A full treatment of the theory of interpolation would carry us beyond the limits of the present discussion. We shall confine ourselves to the case of simple proportional interpolation. The list of references cited below may be consulted by those who wish a detailed treatment of this subject.

Simple Linear Interpolation. The simplest method of interpolation, which may be employed in using tables in which the intervals between the tabled values are small, is by simple proportion, which involves the assumption of a linear relationship between function and argument. We may use the figures set down above as an example. Required: the logarithm of 22.834

$$\begin{aligned} \text{Log } 22.840 &= 1.35870 \\ \text{Log } 22.830 &= 1.35851 \\ \text{Difference} &= \quad .00019 \end{aligned}$$

A difference of .010 in the argument corresponds to a difference of .00019 in the function. The number given, 22.834, exceeds by .004 the smaller of the two numbers tabled in the argument, and we may write

$$\begin{aligned} \text{Log } 22.834 &= 1.35851 + (\tfrac{4}{10} \times .00019) \\ &= 1.35851 + .000076 \\ &= 1.35859 \text{ (rounded off to the fifth decimal place)} \end{aligned}$$

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1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

This operation is facilitated by the use of tables of proportional parts which are given in the margins of many tables of logarithms. Thus, in performing the above interpolation, we should use the marginal table headed 19 (the difference, in a five place table of logarithms, between successive logarithmic values at this point). Of the two columns below the figure 19, that at the left gives the fifth figure of the natural number, the logarithm of which is desired, while that at the right gives the

amount to be added to the logarithm lying just below the desired number. In the present case the fifth figure of the natural number in question (22.834) is 4, hence we add .000076 to the logarithm 1.35851.

The problem of interpolation frequently arises in the handling of simple statistical series, of which the following is an example:

Miles of Railroad in the U. S. 1830–1910

1830	23
1840	2,818
1850	9,021
1860	30,626
1870	52,922
1880	93,267
1890	163,597
1900	193,345
1910	240,438

The approximate mileage in 1877 is desired. Assuming that the increase from year to year during the decade 1870–1880 was by equal absolute increments, we interpolate here by proportional parts.

$$\begin{aligned}\text{Mileage } 1877 &= 52{,}922 + (\tfrac{7}{10} \times 40{,}345) \\ &= 52{,}922 + 28{,}241.5 \\ &= 81{,}163.5, \text{ or } 81{,}163\end{aligned}$$

This method of interpolation makes use only of the pair of observations above and below the value in question, and for ordinary purposes in using prepared tables this method is sufficiently accurate. But sometimes in using tables, and often when the problem of interpolation arises in other fields, a more accurate method is required.

Interpolation by proportional parts or first differences is equivalent to the fitting of a straight line to the two tabled values employed. But sometimes it is preferable to interpolate on the basis of a second degree parabola fitted to three points, a third degree parabola fitted to four points, or a fourth or higher order curve of the potential series. This may be done without actually fitting the curves by the employment of various interpolation formulas, making use of second, third, fourth or higher differences. The use of such formulas is explained in the following references:

Bowley, A. L. *Elements of Statistics* (Fourth Ed.), Chapter IX (P. S. King & Son, London, 1920).

Forsyth, C. H. *An Introduction to the Mathematical Analysis of Statistics*, Chapters II, III (Wiley, N. Y., 1924).

Glover, James W. "Interpolation, Summation and Graduation." (In *Handbook of Mathematical Statistics*, edited by H. L. Rietz, Houghton Mifflin, 1924.)

Jones, D. C. *A First Course in Statistics*, pp. 85–94. (G. Bell & Sons, London, 1921.)

PEARSON, KARL. *Tables for Statisticians and Biometricians*, pp. xiii-xv. (Cambridge University Press, 1914.)

WHITAKER, E. T., and ROBINSON, G. *Calculus of Observations*, Chapters I–IV. (Blackie & Son, London, 1924.)

C

STATISTICAL REPORTS

The final operation in a statistical study is the preparation of the report, and this operation is no less important than the gathering and technical analysis of the data. In a sense it is the most important phase of the study, for the labor that has gone before may be wasted unless the results are presented in telling and effective fashion to those concerned. Accordingly, as much attention should be given to the form of the report as to the preceding parts of the investigation.

Purposes of a Statistical Report. The form of a report must be adapted to the purposes which that report is to serve, and since these purposes will vary no standard form can be laid down as perfect for all purposes. In general, however, a statistical report is designed to perform three main functions, no matter what the subject matter may be. These are:

1. *To present the conclusions to those interested.*

Every study is undertaken to answer certain questions, and a primary purpose of the report must be to present the answers to these questions, to summarize the conclusions to which the investigation has led.

2. *To indicate the conditions under which the study was conducted and to explain the details of the procedure employed.*

If the conclusions which the investigator has reached are to carry weight, they must be accompanied by a complete description of the method employed, an account of the sources of the data utilized, and a statement of all circumstances or conditions which bear upon the validity of the results.

3. *To present the body of the material on which the conclusions are based.*

This material must be presented for the same reason that it is necessary to indicate the conditions under which the study was conducted. All necessary supporting evidence should be in the report, in order that those who wish to confirm the conclusions may have the essential data at hand. Conclusions unsupported by such evidence would carry little weight.

Types of Statistical Reports. Every statistical report serves the three main purposes suggested above, but the emphasis, as among these three purposes, will vary, depending upon the character of the group to which a particular report is addressed. If a comprehensive statistical

study has been made, the conclusions of which are to be presented to persons with varying degrees of technical training, it will be desirable, in general, to prepare two reports.[1] These are:

1. *The scientific record of the investigation.*

This report should present a complete record of the investigation, containing all original data, a record of all calculations, statements as to the degree of error involved in the original observations and in the calculations, copies of all forms employed, etc. This is the basic record which should be available to those desiring to check any detail of the work or to carry the same investigation further.

2. *The general statistical report.*

This is the report designed for administrators, such as department heads or bureau chiefs, men who are not essentially interested in statistical technique, but who have an intelligent understanding of statistical operations and who are familiar with the details of the problems being studied. The primary emphasis in this report is on the conclusions reached by the investigator, but with such a general description of the material employed and the conditions of the investigation as is necessary to a clear understanding of these conclusions.

The general statistical report should be subdivided (in the manner explained in the next section) so that the statement of the problem and of the main conclusions stands out at the beginning, couched in clear, non-technical language. This will permit the non-technical reader, such as a member of a board of directors, readily to grasp the essence of the report.

The scientific record of the investigation should accompany the general statistical report, either as an integral part or as an appendix.

The Form of a Statistical Report. Any standard outline is subject to modification in the preparation of individual reports, but since there are certain general requirements which a report must meet, it is desirable that the report form be correspondingly standardized. The outline which follows applies to both the types of reports mentioned above, with the necessary differences in the degree of detail found under the various headings. If the technical report is to form an integral part of the general statistical report, the introductory sections of the technical report will be modified somewhat.

1. *Statement of the problem.*

The object or objects of an investigation should be clearly stated, on a separate sheet, as the first section of the report. Thus if an in-

[1] This division is based upon Steinmetz' classification of engineering reports (C. P. Steinmetz, *Engineering Mathematics*, pp. 291–292). With the enlarged scope of statistical studies, statisticians may profit from the experience of the engineering profession in this and other matters.

vestigation were undertaken by a large purchaser of copper planning the production of copper roofing materials, in order that manufacturing and purchasing policies might be formulated, the report might open with such a statement as the following:

Purpose: The objects of this investigation are to secure detailed information concerning

(*a*) the production of copper in the United States and in the chief foreign countries, and probable production during the next ten years
(*b*) existing supplies of raw copper
(*c*) international trade in copper
(*d*) consumption of copper and character of current industrial demand for copper; probable demand during the next ten years
(*e*) the general trend of copper prices in the past; seasonal and cyclical fluctuations in copper prices; the relation between changes in the price of copper and movements of the general price level and of the prices of other minerals; probable course of copper prices during the next ten years.

Specifically, the following questions are to be answered:

(*a*) Can the XYZ Corporation count upon securing a steady annual supply of 50,000,000 pounds of copper at prices which will permit the production and sale of copper shingles in competition with other forms of roofing material? [1]
(*b*) Should a contract be made in advance for the delivery of the required copper, and, if so, how long a period should it cover?
(*c*) What season of the year is the most favorable for the purchase of copper?
(*d*) Should a contract be made with a single copper producer, or should purchases be divided?

2. *Statement of conclusions.*

Immediately following the statement of the objects of the investigation, the conclusions should be given, with specific reference to the questions raised in the statement of purposes. The conclusions should be clear and to the point.

3. *Summary of the argument.*

The argument on which the conclusions rest should follow. This should be in the form of a non-technical account of the evidence revealed by the study, with a statement of the weight given the various parts of this evidence by the investigator, and his reasons for reaching the given conclusions.

4. *Outline of procedure, with basic data and major calculations.*

This will constitute the body of the report proper. In the case of the technical, scientific report this will be a detailed and comprehensive

[1] It is assumed, for the purpose of the present illustration, that other studies, dealing with the competition to be expected from other forms of roofing material, have been completed independently of the investigation here described.

account of the data, the methods, the limits of error in data and calculations, etc. A section covering this subject should also be included in the general statistical report, but it will be less detailed than in the technical record.

5. *Appendices.*

Frequently there is no place in the body of the report proper for much of the material which is necessary to a clear understanding of the report or which bears upon the conclusions reached. Such material may properly be included as appendices. As examples of material of this type the following may be mentioned:

(*a*) Data and calculations not given in the body of the report. Sources of all data should be noted.
(*b*) Questionnaire forms and other schedules employed in the study.
(*c*) Definitions and instructions used in the filling out of questionnaires.
(*d*) Subsidiary reports and other documents.

Throughout the report quantitative data and calculations should be shown in tabular form, and the fullest possible use should be made of graphic methods. It goes without saying that close attention should be given to securing perfect clarity of expression and brevity of exposition.

Physical Make-up of a Statistical Report. The physical appearance of the completed report is a final matter of considerable importance, a matter to which the student and beginner is likely to give too little attention. The weight which the report will carry in the minds of those reading it will depend in great part upon its appearance, upon the orderliness of the arrangement, the clearness of the charts, the neatness of the binding and the typing. This is not a matter which can be covered by a set of fixed rules, but a few general suggestions may be made.

1. Paper of a standard form should be employed throughout. The report should be typewritten or, if this is impossible, neatly written out in ink.
2. Chart forms, scales and dimensions should be standardized throughout the report, in so far as possible, and standard practices in graphic presentation should be observed.
3. Careful attention should be given to the lettering of charts. If one is not adept at lettering, and if the services of a draftsman cannot be secured for this purpose, several devices are possible to ensure the neatness of the charts in this respect: (*a*) Scales and titles may be typewritten, (*b*) stencils may be employed, or, (*c*) prepared gummed letters may be purchased.
4. A neat binding should be supplied for the report. In binding care should be taken that the fastening does not prevent the individual sheets from being read.
5. The subject of the report, the name of the chief investigator and other necessary information should be noted on the cover of the report.

II

SUGGESTIONS FOR A STATISTICAL LIBRARY

The following list of books, periodicals and statistical tables is suggested as the basis of a library which should be combined with or convenient to the statistical laboratory. The list is by no means exhaustive, nor will it meet all needs of statistical workers in economics and business. It does include, however, the basic English and American texts and tables and the most useful of the current periodicals. Many important original memoirs and publications in foreign languages have been omitted, though these would be essential in a complete statistical library.

(a) *Books on statistical method, graphic presentation and related subjects.*

Bowley, A. L. *Elements of Statistics.* P. S. King and Son, London. 1920.

Brinton, W. C. *Graphic Methods for Presenting Facts.* The Engineering Magazine Co., New York. 1914.

Brunt, David. *The Combination of Observations.* Cambridge University Press, 1917.

Chaddock, R. E. *Principles and Methods of Statistics.* Houghton Mifflin, Boston. 1925.

Coolidge, J. L. *An Introduction to Mathematical Probability.* Oxford University Press, 1925.

Davenport, C. B. *Statistical Methods, with Special Reference to Biological Variations.* Wiley, New York. 1904.

Davies, G. R. *Introduction to Economic Statistics.* Century, New York. 1922.

Elderton, W. Palin. *Frequency Curves and Correlation.* Layton, London. 1906.

Fisher, Arne. *An Elementary Treatise on Frequency Curves.* Macmillan, New York. 1922.

The Mathematical Theory of Probabilities. Macmillan. New York. 1922.

Fisher, Irving. *The Making of Index Numbers.* Houghton Mifflin, Boston. 1922.

Forsyth, C. H. *Mathematical Analysis of Statistics.* Wiley, New York. 1924.

Haskell, A. C. *Graphic Charts in Business.* Codex Book Co., New York. 1922.

Jerome, Harry. *Statistical Methods.* Harper and Brothers, New York. 1924.

Jones, D. C. *A First Course in Statistics.* Bell, London. 1921.

Karsten, Karl. *Charts and Graphs.* Prentice Hall, New York. 1923.

Kelley, Truman L. *Statistical Method.* Macmillan, New York. 1923.

Kent, Frederick C. *Elements of Statistics.* McGraw Hill, New York. 1924.

Keynes, J. M. *A Treatise on Probability.* Macmillan, New York. 1921.

King, W. I. *Elements of Statistical Method.* Macmillan, New York. 1912.

Lipka, Joseph. *Graphical and Mechanical Computation.* Wiley, New York. 1918.

Manual of Charting. Prentice Hall, Inc., New York. 1924.

Merriman, Mansfield. *The Method of Least Squares.* Wiley, New York. 1897.

Mills, Frederick C. *Statistical Methods Applied to Economics and Business.* Holt, New York. 1924.

Mitchell, W. C. *The Making and Using of Index Numbers.* Part I. Bulletin No. 284. U. S. Bureau of Labor Statistics. Oct., 1921.

Moore, H. L. *Economic Cycles: Their Law and Cause.* Macmillan, New York. 1914.
Forecasting the Yield and the Price of Cotton. Macmillan, New York. 1917.
Generating Economic Cycles. Macmillan, New York. 1923.
Pearl, Raymond. *Medical Biometry and Statistics.* Saunders, Philadelphia. 1923.
Persons, Warren M. *Indices of Business Conditions.* Review of Economic Statistics. Prel. Vol. 1. 1919.
Reinhardt, Charles W. *Lettering.* D. Van Nostrand Co., New York. 1917.
Riegel, Robert. *Elements of Business Statistics.* Appleton, New York. 1924.
Rietz, H. L. (Editor). *Handbook of Mathematical Statistics.* Houghton Mifflin, Boston. 1924.
Rugg, H. O. *Statistical Methods Applied to Education.* Houghton Mifflin, Boston. 1917.
Running, T. R. *Empirical Formulas.* Wiley, New York. 1917.
Secrist, Horace. *Introduction to Statistical Methods.* Macmillan, New York. 1925.
Readings and Problems in Statistical Methods. Macmillan, New York. 1920.
Smith, M. H. *Graphic Statistics in Management.* McGraw Hill, New York. 1924.
Steinmetz, C. P. *Engineering Mathematics.* McGraw Hill, New York. 1917.
Thurstone, L. L. *The Fundamentals of Statistics.* Macmillan, New York. 1925.
Walsh, C. M. *The Measurement of General Exchange Value.* Macmillan, New York. 1901.
The Problem of Estimation. P. S. King and Son, London. 1921.
Weld, L. D. *Theory of Errors and Least Squares.* Macmillan, New York. 1916.
West, Carl J. *Introduction to Mathematical Statistics.* Adams, Columbus. 1918.
Whipple, G. C. *Vital Statistics.* Wiley, New York. 1919.
Whitaker, E. T., and Robinson, G. *The Calculus of Observations.* Blackie and Son, London. 1924.
Wright, T. W., and Hayford, J. F. *Adjustment of Observations.* Van Nostrand, New York. 1906.
Yule, G. Udney. *An Introduction to the Theory of Statistics.* Griffin, London. 1919.
Zizek, Franz. *Statistical Averages.* Holt, New York. 1913.

(b) *Tables to facilitate statistical calculations.*

Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots, Reciprocals. Spon and Chamberlain, New York. 1919.
Carr, G. S. *A Synopsis of Elementary Results in Pure Mathematics.* Francis Hodgson, London. 1886.
Crelle, A. L. *Calculating Tables.* D. Nutt, Berlin. 1898.
Glover, J. W. *Tables of Applied Mathematics.* George W. Wahr, Ann Arbor, Michigan. 1923.
Miner, J. R. *Tables of* $\sqrt{1-r^2}$ *and* $1-r^2$ *for Use in Partial Correlation and Trigonometry.* Johns Hopkins Press, Baltimore. 1922.
Pearson, Karl (Editor). *Tables for Statisticians and Biometricians.* Cambridge University Press. 1914.
(Editor). *Tracts for Computers.* Cambridge University Press, London.
(Editor). *Tables of the Incomplete Gamma Function.* His Majesty's Stationery Office, London. 1922.

(c) *Periodical and occasional publications.*

(1) *Statistical journals.*

Biometrika. Cambridge, England. The University Press.
Journal of the American Statistical Association. Concord, N. H. The Rumford Press.
Journal of the Royal Statistical Society. Published by the Society, London, England.
Metron. Padua, Tipografia Industrie Grafiche Italiane Podova.

(2) *Private publications.*

The Annalist. New York Times Company. New York.
Bradstreet's. The Bradstreet Company. New York.
Commercial and Financial Chronicle. William B. Dana Company. New York.
Dun's Review. R. G. Dun and Company. New York.
Harvard Economic Service, Weekly Letters. Harvard University Committee on Economic Research. Cambridge, Mass.
The Iron Age. Iron Age Publishing Co. New York.
Review of Economic Statistics. Harvard University Committee on Economic Research. Cambridge, Mass.
Statistical Bulletin. Published quarterly by the Standard Daily Trade Service. New York.

(3) *Governmental publications.*

Commerce Yearbook. Bureau of Foreign and Domestic Commerce. Washington, D. C.
Crops and Markets. Published weekly by the U. S. Department of Agriculture, with monthly statistical supplement.
Federal Reserve Bulletin (monthly). Federal Reserve Board. Washington, D. C.
Foreign Commerce and Navigation of the United States (annual). Bureau of Foreign and Domestic Commerce. Washington, D. C.
Fourteenth Census of the United States (11 volumes). Bureau of the Census. Washington, D. C.
Mineral Resources of the United States (annual). U. S. Geological Survey. Washington, D. C.
Monthly Bulletin of Statistics. League of Nations. Geneva.
Monthly Labor Review. U. S. Bureau of Labor Statistics. Washington, D. C. (A valuable pamphlet, *Wholesale Prices of Commodities,* giving the actual and relative prices of the commodities included in the wholesale price index number of the Bureau of Labor Statistics, is issued monthly by that Bureau.)
Monthly Reviews. Federal Reserve Banks.
Monthly Summary of Foreign Commerce of the United States. Bureau of Foreign and Domestic Commerce. Washington, D. C.
Retail Prices. An annual bulletin issued by the United States Bureau of Labor Statistics, Washington, D. C.
Statistical Abstract of the United States (annual). Bureau of Foreign and Domestic Commerce. Washington, D. C.
Survey of Current Business (monthly). United States Department of Commerce. Washington, D. C.
Wholesale Prices. An annual bulletin issued by the U. S. Bureau of Labor Statistics. Washington, D. C.
Yearbook. U. S. Department of Agriculture. Washington, D. C

III

PROBLEMS AND EXERCISES

A. Notes on the Organization of Laboratory Work.

The arrangement of problems in the succeeding pages follows in general the order of treatment of the different subjects covered in the *Text*. An attempt has been made to present a variety of problems and a diversity of problem material from which examples appropriate to given conditions may be selected. The diversity of material will also permit the giving of different problems to individual students or groups of students, a practice which may add to the interest and effectiveness of the teaching.

Problems in section "B" are of an elementary nature designed to refresh the student's mind on the elementary principles of algebra which he will be called upon to use in statistical work or to acquaint him with some of the sources of statistical information.

Examples calling for practice in graphic presentation are in group "C." As these do not require a great deal of time to prepare it is not intended that they be assigned as separate problems. A charting exercise may be given each week at the beginning of the course, along with one or more of the other problems bearing upon the explanation of theory in the class work. The charts should be criticized with reference to the standards for graphic presentation illustrated in the *Text*, pp. 52–59.

Section "D" contains exercises which will afford the student practice in the collection of data by questionnaire and in the subsequent tabulation of such material.

The examples bearing the letter "E" relate to the elementary description of frequency distributions as outlined in Chapters IV and V. It is suggested that the student retain not only a table of his results but also the complete work-sheets of these and other problems. Some of the more advanced problems in later sections refer to those that precede.

The construction of index numbers of prices is illustrated by the series of examples lettered "F." These can be augmented by material readily available in the publications of the Bureau of Labor Statistics and the Department of Agriculture. These problems should be assigned when the class is working on Chapter VI. The simpler examples are intended for practice work in the laboratory hour.

The analysis and combination of time series, explained in Chapters VII, VIII and IX of the *Text*, are illustrated by problems in section

"G." These examples may be varied by substituting material given in section IV of this *Manual*.

Problems dealing with the measurement of correlation are lettered "H" and "I." Explanations of the methods involved are found in the *Text*, Chapters X to XIV.

Section "J" contains exercises on simple probabilities and the calculation of the probable errors of the chief statistical measures. In addition, there are problems calling for the calculation of the moments of a frequency distribution, computation of the criteria of curve type and of certain other measures descriptive of the frequency distribution, and a problem requiring the fitting of the normal curve to a frequency distribution.

Miscellaneous problems, dealing with various business applications of statistical technique, are included in section "K."

The number and character of the problems to be given in connection with a particular course of instruction will depend, naturally, upon the length of the course and the number of hours per week devoted to instruction. To help meet the needs of certain classes of students we have outlined the following assignments, suggesting specific problems which may be given in combination under the stated conditions.

Unless otherwise noted each assignment is designed to cover one week. In order that there may be time for introductory explanations, examinations and reviews only twelve assignments are suggested for each semester.

The short exercises which are given in many of the sections are not specifically assigned. They are designed to be employed at the discretion of the instructor for class room practice.

I. Problems appropriate to a course extending over two semesters.

a. First semester:

Assignment number	Problem number
1	B–5
2	B–6, B–7 and selected problems from C–1
3	C–7 or C–8 and C–10
4	D–1 (1) or D–2 (1), C–2 and C–3
5	E–1, C–4 and C–5
6	E–2, C–6 and C–11
7	E–3
8	E–4 and E–5
9	E–6 and E–7
10	E–9
11	F–3 or F–4
12	F–6 and F–9 or F–10

b. Second semester:

Assignment number	Problem number
1	G–1
2	G–4 and G–5
3	G–7
4	G–8
5	G–9
6	G–14
7	K–1 or K–6
8	H–2 and H–3
9	H–4
10	H–5 or H–6
11	H–7
12	H–8

II. Problems appropriate to a general course in statistics designed to cover the field in a less intensive fashion in a single semester.

Assignment number	Problem number
1	B–5 or B–6, and B–7
2	Selected problems from C–1 and C–2, C–3
3	C–4, C–5 and C–11
4	D–1 (1) and E–1
5	E–4 and E–5
6	E–6 and E–7
7	F–3 or F–4
8	F–6 and F–9 or F–10
9	G–4 and G–5
10	G–9
11	H–2 and H–3
12	H–7

III. Problems appropriate to an advanced course in statistics covering one semester. (The course indicated under I above is assumed to be a pre-requisite to this course.)

Assignment number	Problem number
1	H–10
2	H–11 or H–12
3 (Two weeks)	H–13
4 (Two weeks)	I–1 or I–2
5	J–4 and J–6
6	J–7
7	J–8
8	J–9, J–10, J–11, J–12, J–13 and J–14
9	K–3
10	K–4

Laboratory Sets. In the preparation of laboratory reports attention should be given to the general principles relating to statistical reports which have been outlined above (*Manual*, pp. 31–34). Standard laboratory sets, consisting of folders for the binding of reports and coördinate and logarithmic plotting paper appropriate to the problems in this *Manual*, have been prepared by the authors. These may be obtained by order from the Codex Book Company, 461 Eighth Avenue, New York.

In addition to the plotting paper and folders which are included in these prepared sets, the following equipment will be found useful to the student in connection with the graphic part of statistical practice:

Drawing board
T-square
45° triangle
Flexible curve or French curve
Drafting pen
Lettering pen
Rule or straight edge

In performing the arithmetic operations outlined in connection with any of the problems in this Manual, *it is essential that a formal procedure employing appropriate work sheets be followed. Students should give careful attention to the planning of such work sheets.*

B. Introductory Exercises

1. Simplify the following expressions:

(1). $(3ax + b) + (2ax - b)$
(2). $(ay + 4b) - (2ay - b)$
(3). $2a(y + b) - 2y(a - b)$
(4). $(3a - b)(a + 2b)$
(5). $(16 + 8b + b^2) \div (4 + b)$
(6). $(2a + 3b)^2$
(7). $(a + b)^3$
(8). $\sqrt{9a^2 - 42ab + 49b^2}$
(9). $\sqrt[3]{64a^9}$
(10). $(32a^{15})^{\frac{1}{5}}$

2. Solve the following simultaneous equations:

(1). $3a + 2b = 23$
$7a + 5b = 56$
(2). $8a - 2b = 22$
$3a + 5b = 2.5$
(3). $2a - 10b = 1372$
$a + 2b = 425.6$

3. Verify the following statements by consulting the table of logarithms in this *Manual:*

$$\log .4896 = 9.68984 - 10$$
$$\log 4.896 = .68984$$
$$\log 48.96 = 1.68984$$
$$\log 489.6 = 2.68984$$

$$\text{anti-log } 2.66096 = 458.1$$
$$\text{anti-log } 1.66096 = 45.81$$
$$\text{anti-log } 1.24581 = 17.612$$
$$\text{anti-log } 9.83686 - 10 = .68685$$

4. By the use of logarithms perform the following operations:

(1). $(83.9)\,(43.2)$
(2). $(.986)\,(5.89)$
(3). $(.0567)\,(.0049)$
(4). $7389 \div 53.89$
(5). $.0983 \div 5.962$
(6). $(78.56)^5$
(7). $(325.6)^7$
(8). $(9862)^{\frac{1}{3}}$
(9). $\sqrt{(289)\,(43.2)}$
(10). $\sqrt[5]{791.8}$
(11). $\dfrac{(83.96)\,(8.396)^2}{\sqrt{83.96}}$
(12). $\dfrac{(9763.8)^{\frac{1}{2}}}{3(8.777)^2}$
(13). $\sqrt[8]{5^2 \cdot 10^3 \cdot 15^2 \cdot 20}$
(14). $\sqrt[10]{3 \cdot 6^2 \cdot 9^4 \cdot 12^2 \cdot 15}$

(Problems similar to these should be given so that the student may gain facility in the manipulation of logarithms. The accuracy of the results may be checked by going through the operations on the calculating machine.)

If the laboratory is equipped with adding and calculating machines the first exercises should be devoted to familiarizing the student with machine calculation. To this end the instructor can take advantage of the sets of practice problems which manufacturers of these machines provide.

5. The following information is to be gathered from the proper sources and tabulated in the order indicated. Sources from which the data are obtained are to be given in all cases.

(1). Rate of interest on call loans, New York, high and low, for each day of the current week.

(2). Rate of interest on call loans, New York, high and low, for the current week, the week preceding and the week corresponding to the current week last year.

(3). Rate of interest on time loans, 60–90 day commercial paper, choice, double name, for the same periods.

(4). Rediscount rate on 60–90 day commercial paper at the Federal Reserve Bank of your district on the first Saturday of the current month and on the first Saturday of last month.

(5). Bank of England rate, same dates.

(6). Bank clearings, U. S. (excluding N. Y. City) for last month, the preceding month, and for the month corresponding to last month in the preceding year.

(7). Debits to individual accounts in New York City for the same periods.

(8). The ratio of total reserves to deposit and Federal Reserve note liabilities, for all Federal Reserve Banks combined, for the same periods.

(9). Values of imports and exports of the U. S. for the last two months for which figures are available, and for the same months of last year.

(10). Demand sterling N. Y. (i.e., foreign exchange on London) for each day of the current week.

(11). The Bureau of Labor Statistics index number of wholesale prices, all commodities, for the last twelve months; for the last calendar year and the three preceding calendar years.

(12). Price of cotton, middling upland, spot, N. Y., last Saturday, the same day a week ago, and last Saturday a year ago.

(13). Price of wheat, No. 2, red winter, Chicago, same dates.

(14). Price of Bessemer steel billets, Pittsburgh, same dates.

(15). Number of shares of stock sold on the N. Y. Stock Exchange for last month, the preceding month, and for the month corresponding to last month in the preceding year.

(16). U. S. Steel common stock:

a. Sales for the current week
b. High and low prices for the current week
c. High and low prices for the current year, with dates

(17). U. S. Steel preferred stock: same data as for common stock.

6. Select one of the following commodities for study:

Pig iron	Zinc	Coal (Anthracite)	Petroleum
Copper	Coal (Bituminous)	Coke	Gasoline
			Portland Cement

Obtain the following information concerning the commodity selected:

(1). Unit in which sold.

(2). Chief grades (if any) with a brief description of each.

(3). Prices:

a. Average annual price at wholesale since 1913.
b. Average monthly price at wholesale for the last twelve months. (If average monthly prices are not available the price on the last day of each month may be given.)

(4). Amounts and values of commodity imported and exported last month, last year, and in 1913.

(5). Amounts produced in the U. S. each year since 1913.

(6). Number of establishments and number of persons engaged in the manufacture or production of the given commodity in 1921, 1919, 1914.

Instructions:

(*a*) In quoting prices, indicate the commodity unit, the grade and the market for which prices are given.

(*b*) Sources from which price quotations and other figures are taken are to be given in all cases.

(*c*) Any notes in the original sources which qualify or explain the figures quoted should be repeated in the report.

(This problem can be varied by including additional commodities and making suitable changes in the questions. Agricultural commodities should be included where these are of local interest.)

7. Select one of the following publications for intensive study and draw up a report showing an appropriately classified list, arranged in alphabetical order, of all business facts that are regularly reported in the current numbers. Is an annual statistical number published? If so, what additional facts are contained in it? How long has it been published? When does it regularly appear? What supplements (if any) are published?

Bradstreet's, Dun's Review, The Annalist, The Wall Street Journal, The Journal of Commerce, The Commercial and Financial Chronicle, The Survey of Current Business, The Iron Age, Automotive Industries, the local daily papers.

C. Graphic Presentation.

1. Present the following equations graphically:

(1). $y = a + bx$

(*a*) when $a = 1$ and $b = 2$

(*b*) when $a = 0$ and $b = 2$

(*c*) when $a = -2$ and $b = 2$

(*d*) when $a = 5$ and $b = -2$

(*e*) when $a = 5$ and $b = 5$

What does the value of "a" determine? of "b"? What effect do negative signs have on the position of the line?

(2). $y = a + bx + cx^2$

(*a*) when $a = 5$; $b = 3$ and $c = 2$

(*b*) when $a = 5$; $b = 3$ and $c = -2$

(*c*) when $a = -5$; $b = -3$ and $c = 2$

(*d*) when $a = -5$; $b = -3$ and $c = -2$

(3). $y = \dfrac{1}{a + bx}$

(*a*) when $a = 0$ and $b = 1$

(*b*) when $a = 2$ and $b = 3$

(4). $y = ax^b$

(a) when $a = 1$ and $b = 1$
(b) when $a = 1$ and $b = 2$
(c) when $a = 2$ and $b = 1.2$
(d) when $a = \frac{1}{4}$ and $b = 2$
(e) when $a = 2$ and $b = -2$

(5). $y = a^x$

(a) when $a = 2$
(b) when $a = 1.5$

(6). $y = a(b)^x$ Compound int.

(a) when $a = .5$ and $b = 1.5$
(b) when $a = 2$ and $b = 2$

2. Graphs of parabola (natural numbers and logarithms):

(1). Construct a graph on the natural scale, of the equation

$$y = 4x^2 \qquad \text{(I)}$$

The graph should be accompanied by a working table showing sample values of x and corresponding values of y. Values of the function (y) corresponding to negative as well as positive values of x should be shown.

(2). Construct a graph on the natural scale of the logarithm of the above equation (i.e., $\log y = \log 4 + 2 \log x$). (II)
In plotting the equation $\log y$ is plotted against $\log x$. The graph should be accompanied by a working table showing sample values of x, values of $\log x$, $\log 4$, and $\log y$.

(3). What is the slope of the line thus plotted? To what in equation I does the slope correspond?

(4). What is the value of the y-intercept? To what in equation I does the y-intercept correspond?

3. Graphs of exponential curve (natural numbers and logarithms):

(1). Construct a graph on the natural scale, of the equation

$$y = 3(2^x) \qquad \text{(III)}$$

The graph should be accompanied by a working table showing sample values of x and corresponding values of y.

(2). Construct a graph of the logarithm of the above equation [i.e., $\log y = \log 3 + x \ (\log 2)$]. (IV)
In plotting the equation, $\log y$ is plotted against x.

The graph should be accompanied by a working table showing sample values of x, values of $\log 3$, $\log 2$, and $\log y$.

(3). What is the slope of the line thus plotted? To what in equation III does the slope correspond?

(4). What is the value of the y-intercept? To what in equation III does the y-intercept correspond?

4. On a sheet of semi-logarithmic paper construct graphs of the following equations:

(1) $y = (2)^x$
(2) $y = 2(2)^x$
(3) $y = -2(3)^x$
(4) $y = 2(3)^{-x}$

5. On a sheet of double logarithmic paper construct graphs of the following equations:

(1) $y = x^2$
(2) $y = 4x^2$
(3) $y = x^3$
(4) $y = \frac{1}{x}$

Two-cycle paper should be employed. If the two axes are scaled from .1 to 10, the four equations may be plotted on a single chart. Each graph should be labelled.

6. On arithmetic cross section paper plot the data relating to the growth of railroad mileage in the United States given in the table below. Connect these points with straight lines. When was the rate of growth most rapid?

Plot the same curve on semi-logarithmic paper. Account for the difference in the appearance of the curves.

Miles of Railroad in the U. S., 1830–1920

1830	23
1840	2,818
1850	9,021
1860	30,626
1870	52,922
1880	93,267
1890	163,597
1900	193,345
1910	240,438
1920	252,844

Source: *Statistical Abstract of the U. S.*

7. The trend of gasoline consumption in the United States from 1918 through 1924 can be represented by the straight line

$$y = 5023.7 + 770.3x$$

where

y is the annual consumption in millions of gallons
x is the time in years, with the origin at 1921

(That is, for 1921, $x = 0$, for 1922, $x = +1$, for 1920, $x = -1$, etc.)

Plot this equation on ordinary arithmetic paper. Label the axes, designate the scales and give the chart an appropriate title.

What is the slope of this line?

What is the average annual increase in gasoline consumption as described by the line of trend?

8. The average relationship in the different states between the number of people making out income tax returns and the number of automobiles registered in 1922 can be expressed by the straight line

$$y = 64.50 + 1.48x$$

where

x is the number of people, in thousands, making out tax returns, and

y is the registration of automobiles in thousands.

Plot this line on ordinary cross section paper. Label the line and the axes and indicate the scales.

If a certain state reported that 45,500 people made out income tax returns in 1922, what estimate would you make of the number of automobiles then registered in that state?

9. The trend of annual production of a certain commodity from 1911 to 1919 can be represented by the equation $y = 2765x^{1.20}$ where y is the total annual production in thousands of tons and x is the time in years, with origin at 1910 (i.e. for 1911, x is 1; for 1912, x is 2, etc.).

Calculate values for y for each year from 1911 to 1919 and plot them on simple arithmetic coördinate paper.

From this equation what would you estimate the production to be for the year 1920?

Plot the calculated values for y on semi-logarithmic paper.

10. The following figures represent the monthly sales of F. W. Woolworth and Company, in thousands of dollars:

Month	1919	1920	1921	1922	1923	1924
January	$ 7,128	$ 8,491	$ 8,336	$ 9,517	$11,048	$12,134
February	7,189	8,170	9,138	10,080	11,234	13,431
March	9,120	10,853	11,831	11,847	15,782	15,903
April	9,139	10,744	10,963	13,439	13,940	17,257
May	9,924	11,320	11,203	12,881	14,799	17,075
June	8,542	10,818	10,741	12,433	14,912	15,485
July	8,718	11,283	10,744	12,557	14,002	15,950
August	9,553	11,070	11,641	12,958	14,964	16,927
September	9,046	10,957	11,325	13,508	14,775	16,526
October	10,743	13,242	14,408	15,774	18,085	20,393
November	10,985	12,428	13,107	14,835	17,283	19,371
December	19,375	21,522	24,191	27,463	32,626	35,021

Construct an ordinary arithmetic chart based upon these figures. The figures plotted should be presented in a table accompanying the chart.

Briefly comment on the trend of sales over the six year period, pointing out any relation you may find between the volume of sales and the seasons of the year.

11. The Essex Soap Company has the following record of sales and advertising expense for the period 1913–1924, inclusive:

	Total Sales	Sales Western Territory	Advertising Expense Western Territory	Sales Eastern Territory	Advertising Expense Eastern Territory
1913	$ 400,000	$100,000	$ 5,000	$300,000	$12,000
1914	420,000	100,000	5,000	320,000	12,000
1915	440,000	110,000	5,500	330,000	13,000
1916	550,000	140,000	6,000	410,000	13,500
1917	800,000	180,000	6,000	620,000	15,000
1918	900,000	200,000	8,000	700,000	16,000
1919	900,000	210,000	8,000	690,000	16,500
1920	1,000,000	250,000	8,500	750,000	17,500
1921	600,000	160,000	10,000	440,000	15,000
1922	650,000	180,000	10,000	470,000	16,000
1923	750,000	200,000	8,000	550,000	16,500
1924	680,000	180,000	8,000	500,000	16,500

(1.) Plot the above figures on arithmetic cross section paper. The base line should be zero. Use the same scale throughout. Plot each curve separately from the base line. Distinguish between the different lines by using different colored inks or different kinds of broken lines.

(2.) Plot the above figures on a semi-logarithmic ("ratio") chart. Use paper having two cycles vertically. Plot each curve separately from the base line (i.e., do not attempt to show component parts).

(3.) On a separate sheet give a brief description of the relation between sales and advertising expense as revealed by a study of the two charts. By years, compare the rates of increase or decrease in sales with the rates of increase or decrease in advertising expenses. Which chart brings out the essential facts most clearly?

12. The XYZ Corporation has the following record of gross revenue, expenses and net revenue for the period 1911–1924:

	Gross Revenue	Expenses	Net revenue
1911.....	$4,375,000	$3,950,000	$425,000
1912.....	4,500,000	3,975,000	525,000
1913.....	4,750,000	4,185,000	565,000
1914.....	4,700,000	4,200,000	500,000
1915.....	3,900,000	4,200,000	*300,000* Deficit
1916.....	4,800,000	4,200,000	600,000
1917.....	5,800,000	4,950,000	850,000
1918.....	6,050,000	5,400,000	650,000
1919.....	6,200,000	5,350,000	850,000

	Gross revenue	Expenses	Net revenue
1920	5,100,000	4,900,000	200,000
1921	4,100,000	4,050,000	50,000
1922	5,500,000	4,800,000	700,000
1923	5,200,000	4,700,000	500,000
1924	5,600,000	4,900,000	700,000

Figures are for fiscal years ending June 30.

Construct a chart on the natural scale based upon the above figures.

13. Draw a series of vertical bar charts to show the relative importance of each of the New England states in respect to area, population and miles of railroad, as given in the following table:

State	Area Square miles (1920)	Population (1920)	Miles of Railroad (1919)
Maine	33,040	768,014	2,294
New Hampshire	9,305	443,083	1,255
Vermont	9,565	352,428	1,077
Massachusetts	8,315	3,852,356	2,126
Rhode Island	1,250	604,397	212
Connecticut	4,990	1,380,631	1,022
Total	66,465	7,400,909	7,986

14. Prepare a series of bar charts showing the changes that have taken place in the relative importance of the different elements making up the manufacturer's price for one of the commodities for which the data are presented below:

(1). *Distribution of the Dollar the Wholesaler Pays to the Manufacturer of Men's Suits*

	1913	1920	1921
	cents	cents	cents
Cost of raw material	26.60	30.60	26.70
Cost of trimmings	17.80	13.50	14.80
Manufacture	29.30	28.50	34.20
Overhead	19.00	21.60	22.10
Profit	7.30	5.80	2.20

(2). *Distribution of the Dollar the Wholesaler Pays to the Manufacturer of Corn Flakes*

	1913	1916	1917	1918	1919	1920	1921
	cents	cents	cents	cents	cents	cents	cents
Cost of raw material...	36.19	50.51	59.34	59.28	47.94	57.49	39.19
Manufacture.....	9.09	10.65	10.66	9.64	8.99	12.91	11.11
Selling.....	15.62	15.52	6.95	2.40	5.79	6.07	10.00
Advertising.....	17.02	7.77	3.83	2.69	3.43	4.60	6.16
Transportation.....	10.28	11.93	10.05	8.60	8.56	9.34	12.51
Taxes.....	4.33	3.05	4.54	10.22	11.42	5.16	9.57
Profit.....	7.47	.57	4.63	7.17	13.87	4.43	11.46

Source: *Report of the Joint Commission of Agricultural Inquiry.*

15. In an advertisement the General Electric Company reports the following distribution of its gross income from 1918 to 1922, inclusive:

Material, Supplies, Depreciation and Losses.....	42.2%
Salaries and Wages.....	40.9
Cash Dividends.....	4.8
Surplus.....	4.7
Taxes Paid.....	3.6
Transportation, Telephone and Telegraph.....	2.7
Interest on Borrowed Money.....	1.1
	100.0%

Prepare a chart to bring out the relative importance of the different items given above.

16. From the following figures construct charts to show the geographical distribution of population and income and to permit comparison of the geographical districts with respect to per capita income:

Population and Estimated Income in the United States by Geographical Divisions, 1919

Geographical Division	Total income (thousands)	Population Jan. 1, 1920
United States.....	$66,252,601	105,710,620
New England.....	5,358,511	7,400,909
Middle Atlantic.....	17,424,981	22,261,144
East North Central.....	14,681,293	21,475,543
West North Central.....	7,650,410	12,544,249
South Atlantic.....	6,473,303	13,990,278
East South Central.....	3,239,774	8,893,307
West South Central.....	5,030,057	10,242,224
Mountain.....	1,962,585	3,336,101
Pacific.....	4,431,687	5,566,871

Source: *Distribution of Income by States in 1919*, National Bureau of Economic Research.

17. A study of the ABC corporation is to be made. From their annual statements the following figures are obtained:

	1918	1919	1920	1921	1922	1923	1924
Stock outstanding [1]	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000
Bonds [1]	15,000	15,000	15,000	12,000	12,000	12,000	12,000
Surplus [1]	4,000	8,000	10,000	9,000	11,000	12,000	15,000
Working capital [1]	20,000	22,000	26,000	25,000	30,000	42,000	35,000
Net for dividends [1]	8,000	10,000	6,000	4,000	4,000	5,000	3,000
Average price of stock	80	100	110	80	90	120	110

[1] In thousands.

Present this information graphically. Point out briefly the important trends, and the changes that have taken place during this period.

18. In a pamphlet the Travelers Insurance Company states that there are ten million accidents in the United States each year. Of every 100 men who are killed or injured 17 are victims of home accidents, 20 are in automobile mishaps, 16 are hurt in sports and recreations, 11 are walking when fate overtakes them, 31 are victims of occupational accidents and 5 are hurt in miscellaneous ways.

Present this information in the form of a "pie" or circle chart. (Let the area of the circle represent 100 and subdivide to indicate the relative importance of the different causes.)

Draw a series of bar diagrams based upon the same information. Which method do you consider more effective?

19. The ABC Corporation has the following record of costs per unit of product, sub-divided, for the first eight months of 1925:

	Jan.	Feb.	March	April	May	June	July	Aug.
Cost, Mat. A	$17.50	$18.20	$18.50	$19.70	$20.00	$19.00	$19.00	$19.00
Cost, Mat. B	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Cost, Misc. Mat.	6.00	6.60	6.70	7.00	6.80	6.60	7.00	7.20
Cost, Labor	42.00	45.00	44.00	46.00	46.50	45.00	50.00	53.00
Overhead Exp.	65.00	67.00	62.00	65.00	68.00	70.00	77.00	80.00
Total Cost (per unit)	$140.50	$146.80	$141.20	$147.70	$151.30	$150.60	$163.00	$169.20

Construct a chart showing the variations in the cost of the standard unit and of the items making up that cost. (Component parts are to be differentiated by shading or by the use of colors. Total material costs are to be distinguished from other costs. See *Text* p. 47.)

Draw up a memorandum for the factory manager calling his attention to the chief points brought out by the chart and making such suggestions as may be called for.

20. The F. W. Dodge Corporation gathers information relative to the extent of building activity in the thirty-six Eastern States. Some idea of the volume of potential construction can be obtained from the following figures:

Year	Contemplated work (*a*)	Contracts awarded (*b*)	Ratio (*b*:*a*)
1918	$4,046,100,000	$1,944,300,000	.48
1919	5,013,900,000	2,969,300,000	.59
1920	5,655,200,000	2,951,700,000	.52
1921	5,459,100,000	2,710,800,000	.50
1922	7,796,400,000	3,848,700,000	.49
1923	7,459,900,000	3,990,500,000	.53
1924	6,652,300,000	4,481,800,000	.67

The term "contemplated work" means plans prepared and reported by architects.

"Contracts awarded" means the actual contracts for work started during the current year.

Construct a chart to bring out the facts revealed by these figures, and interpret the chart in the light of business conditions prevailing during this period.

21. The following table gives the monthly sales quotas and the actual sales for three salesmen of the Blank Manufacturing Corporation during the past year.

Construct cumulative charts showing the relation between sales quotas and actual sales for each man (*Text* p. 48).

Construct Gantt Progress Charts showing the situation as of October 31st for all three salesmen (*Text* p. 50).

	Jones		Smith		Brown	
	Quota	Sales	Quota	Sales	Quota	Sales
Jan....	$ 8,000	$ 8,100	$6,000	$ 5,500	$ 7,000	$5,000
Feb. ..	8,500	8,400	6,500	6,000	7,000	5,500
Mar...	10,000	9,500	7,500	7,000	8,000	6,000
Apr. ..	11,000	10,500	8,000	7,500	8,000	6,000
May ..	11,000	10,000	8,500	8,500	8,500	6,200
June ..	10,000	10,000	8,000	8,100	8,500	6,500
July...	9,000	9,500	7,500	7,800	7,000	6,000
Aug. ..	9,000	9,000	7,000	7,500	6,500	6,100
Sept...	9,500	9,200	7,500	8,000	7,000	6,500
Oct....	10,000	10,000	8,500	9,000	7,500	6,700
Nov...	10,000	9,400	8,500	9,000	7,500	7,000
Dec. ..	12,000	11,000	9,000	10,500	10,000	8,000

22. According to figures collected by Bureau of Labor Statistics in 1920, the average family expenditure in Los Angeles, California, was distributed according to the following table. Present this material graphically.

Item of expenditure	Percent of total
Food	35.8
Clothing	14.9
Housing	13.4
Fuel and light	3.1
Furniture and furnishings	5.1
Miscellaneous and savings	27.7
Total	100.0

D. Collection and Tabulation of Statistical Material.

1. A survey of the cost of laying bricks was made by the Bureau of Labor Statistics in 1923. The data given below are taken from the results of this study published in the *Monthly Labor Review*, November, 1924.

(1). Draw up a summary table showing, for the different cities,

the average rate paid per hour
the average number of bricks laid per hour
the average cost of laying 1000 bricks.

(2). Construct frequency tables classifying all the bricklaying jobs, for which data were collected, according to

the rate paid per hour to bricklayers
the number of bricks laid per hour
the cost of laying 1000 bricks.

Labor Productivity and Cost in Specified Building Trades, by Cities
Bricklaying

City and job number	Union, nonunion or mixed	Rate per hour	Number of bricks laid per hour	Cost of laying 1000 bricks
Atlanta, Ga.:				
1	Union	$1.125	161.5	$6.97
2	"	1.250	123.1	10.15
3	"	1.250	308.6	4.05
4	"	1.250	299.2	4.18
5	Nonunion	.900	160.0	5.63
6	"	.900	159.7	5.64
7	"	.900	159.3	5.65
8	Mixed	1.000	250.0	4.00
9	"	1.000	144.7	6.91
10	"	1.000	138.0	7.25
11	Nonunion	1.000	161.0	6.21
12	"	1.000	99.9	10.01
13	"	1.000	125.8	7.95

Continued on next page.

City and job number	Union, Nonunion or mixed	Rate per hour	Number of bricks laid per hour	Cost of laying 1000 bricks
14	Nonunion	1.000	149.0	6.71
15	"	1.000	129.3	7.73
16	Mixed	.800	148.1	5.54
17	Union	1.125	235.6	4.78
18	"	1.125	328.5	3.42
19	"	1.250	293.1	4.26
20	"	1.250	201.2	6.21
21	Mixed	1.250	160.0	7.81
22	"	1.250	175.6	7.12
23	Nonunion	1.000	151.4	6.61
Average		$1.050	185.3	$6.30
Birmingham, Ala.:				
1	Union	$1.125	250.0	$4.50
2	"	1.250	243.8	5.13
3	"	1.125	255.3	4.41
4	"	1.125	229.1	4.91
5	"	1.125	203.4	5.53
6	"	1.125	248.5	4.53
7	"	1.125	240.0	4.69
8	"	1.250	258.0	4.84
Average		$1.160	241.0	$4.82
Chattanooga, Tenn.:				
1	Nonunion	$1.250	318.9	$3.92
2	Union	1.500	237.0	6.33
3	"	1.500	218.2	6.87
4	"	1.500	250.4	5.99
5	Nonunion	1.250	242.6	5.69
6	Union	1.500	147.5	10.17
7	"	1.500	151.6	9.89
8	"	1.500	246.0	6.10
9	"	1.500	202.1	7.42
10	"	1.500	270.0	5.56
11	"	1.500	203.0	7.39
Average		$1.460	226.1	$6.85
New Orleans, La.:				
1	Union	$1.100	143.3	$7.68
2	"	1.000	215.0	4.65
3	"	1.000	210.0	4.76
4	"	1.000	220.6	4.53
5	"	1.000	217.8	4.59
6	"	1.000	210.0	4.76
7	"	1.000	207.7	4.81
Average		$1.010	203.5	$5.11

City and job number	Union nonunion or mixed	Rate per hour	Number of bricks laid per hour	Cost of laying 1000 bricks
Norfolk, Va.:				
1	Nonunion	(1)	341.2	(1)
2	"	(1)	223.6	(1)
3	Union	$1.375	194.4	$7.07
4	"	1.375	231.7	5.93
5	"	1.375	230.4	5.97
6	"	1.375	198.0	6.94
7	"	1.375	286.3	4.80
8	"	1.375	187.0	7.35
9	"	1.375	225.0	6.11
10	"	1.375	200.0	6.88
Average		$1.375 [2]	231.8	$6.38 [2]
Boston, Mass.:				
1	Union	$1.250	100.5	$12.44
2	"	1.250	93.1	13.43
3	"	1.250	118.0	10.56
4	"	1.250	86.9	14.38
5	"	1.250	90.1	13.87
Average		$1.250	97.7	$12.94
New York, N. Y.:				
1	Union	$1.875	146.1	$12.83
2	"	1.875	150.0	12.51
3	"	1.500	210.1	7.14
4	"	1.750	103.9	16.84
5	"	1.875	146.4	12.81
6	"	1.750	150.0	11.67
7	"	1.750	151.9	11.52
8	"	1.750	181.2	9.66
9	"	1.750	115.3	15.17
10	"	1.750	190.9	9.17
11	"	1.750	143.0	12.24
12	"	1.750	202.1	8.66
Average		$1.760	157.6	$11.69
Philadelphia, Pa.:				
1	Union	$1.500	128.0	$11.72
2	"	1.500	103.4	14.51
3	"	1.500	152.9	9.81
4	"	1.500	115.9	12.94
5	"	1.500	96.2	15.59
6	"	1.500	110.5	13.57
7	"	1.500	117.9	12.72
8	"	1.500	117.9	12.72
9	"	1.500	145.5	10.31
10	"	1.500	150.1	9.99

[1] Not reported. [2] Not including two jobs not reported.

Continued on next page.

City and job number	Union, nonunion or mixed	Rate per hour	Number of bricks laid per hour	Cost of laying 1000 bricks
11	Union	1.600	122.2	13.09
12	"	1.600	122.2	13.09
13	"	1.750	112.5	15.56
14	"	1.500	131.3	11.42
Average		$1.530	123.3	$12.65
Chicago, Ill.:				
1	Nonunion	$1.500	151.7	$9.89
2	"	1.500	100.0	15.00
3	"	1.550	134.1	11.56
4	Union	1.500	150.3	9.98
5	"	1.500	173.2	8.66
6	"	1.600	122.7	13.04
7	"	1.500	125.0	12.00
8	"	1.500	135.0	11.11
9	"	1.500	140.0	10.71
10	"	1.580	207.4	7.62
11	"	1.650	143.3	11.51
12	"	1.650	241.2	6.84
13	"	1.630	99.7	16.35
14	"	1.650	137.7	11.98
15	"	1.500	198.5	7.56
16	Mixed	1.650	150.0	11.00
17	Union	1.500	202.7	7.40
18	Mixed	1.500	257.8	5.82
19	"	1.600	144.0	11.11
20	"	1.680	191.4	8.78
21	"	1.600	143.3	11.16
22	"	1.630	151.1	10.79
23	"	1.700	124.2	13.69
24	"	1.500	139.3	10.77
Average		$1.570	156.8	$10.60
Cincinnati, O.:				
1	Union	$1.500	123.8	$12.12
2	"	1.500	150.3	9.98
3	"	1.500	199.8	7.52
4	"	1.250	150.0	8.34
5	"	1.250	151.5	8.25
6	"	1.250	125.7	9.95
7	"	1.500	119.3	12.57
8	"	1.500	93.7	16.01
9	"	1.500	100.4	14.94
10	"	1.500	100.0	15.00
Average		$1.430	131.5	$11.47
Cleveland, O.:				
1	Union	$1.500	153.4	9.78
2	"	1.500	92.0	16.31
3	"	1.500	163.4	9.18

Continued on next page.

City and job number	Union, nonunion or mixed	Rate per hour	Number of bricks laid per hour	Cost of laying 1000 bricks
4	"	1.500	208.6	7.19
5	"	1.500	174.9	8.58
6	"	1.500	119.9	12.51
7	"	1.500	168.9	8.88
8	"	1.500	150.8	9.95
9	"	1.500	140.0	10.71
10	"	1.500	131.3	11.43
11	"	1.500	145.9	10.29
12	"	1.500	132.0	11.37
13	"	1.500	167.5	8.96
14	"	1.500	120.0	12.50
Average		$1.500	147.8	$10.55
Denver, Colo.:				
1	Union	$1.500	266.4	$5.63
2	"	1.500	275.2	5.45
3	"	1.500	155.3	9.66
4	"	1.500	175.0	8.57
5	"	1.500	163.2	9.19
6	"	1.500	277.8	5.40
7	"	1.500	146.0	10.28
8	"	1.500	300.7	4.99
9	"	1.500	203.8	7.36
10	"	1.500	200.3	7.49
11	"	1.500	195.0	7.69
12	"	1.500	324.5	4.65
13	"	1.500	213.3	7.03
14	"	1.500	153.8	9.75
15	"	1.500	200.0	7.50
16	"	1.500	277.4	5.41
17	"	1.500	156.5	9.58
18	"	1.500	138.5	10.83
Average		$1.500	212.4	$7.58
Detroit, Mich.:				
1	Union	$1.600	123.6	$12.94
2	"	1.510	192.8	7.83
3	"	1.510	130.2	11.60
4	"	1.600	149.6	10.69
5	"	1.500	137.0	10.95
6	"	1.520	106.6	14.26
7	"	1.560	192.5	8.10
8	"	1.520	179.2	8.48
9	"	1.610	107.8	14.94
10	"	1.520	148.0	10.27
11	"	1.520	205.0	7.42
12	"	1.520	159.6	9.52
13	"	1.520	162.5	9.35
14	"	1.600	173.6	9.21
15	"	1.570	141.4	11.10
Average		$1.545	154.0	$10.44

City and job number	Union, nonunion or mixed	Rate per hour	Number of bricks laid per hour	Cost of laying 1000 bricks
Indianapolis, Ind.:				
1	Union	$1.350	89.3	$15.13
2	"	1.350	85.3	15.82
3	"	1.350	106.1	12.72
4	"	1.375	105.5	13.04
5	"	1.375	79.6	17.28
6	"	1.375	107.5	12.79
7	"	1.375	112.6	12.21
8	"	1.375	90.2	15.24
9	"	1.375	90.2	15.24
10	"	1.375	90.2	15.24
Average		$1.368	95.7	$14.47
Minneapolis, St. Paul, Minn.:				
1	Union	$1.125	143.7	$7.83
2	Nonunion	1.125	155.0	7.26
3	"	1.125	158.4	7.10
4	Mixed	1.125	103.2	10.90
5	"	1.125	226.0	4.98
6	Union	1.250	146.6	8.53
7	"	1.500	167.5	8.96
8	Mixed	1.250	162.5	7.69
9	"	1.250	225.0	5.56
10	"	1.250	232.3	5.38
11	Union	1.250	171.3	7.07
12	Mixed	1.125	201.5	5.58
13	"	1.125	201.5	5.58
14	"	1.375	300.0	4.58
15	Union	1.250	154.3	8.10
16	"	1.250	286.8	4.36
17	Mixed	1.188	87.5	13.58
18	Union	1.250	238.7	5.24
19	"	1.250	275.0	4.55
20	Mixed	1.125	200.0	5.65
21	Union	1.125	220.0	5.11
Average		$1.211	193.2	$6.84

2. (1). From the following data, collected by the Bureau of Labor Statistics in 1923, construct a summary table to show, for the different cities,

the average rate per hour paid for plastering
the average area plastered per hour
the average cost of plastering per square yard.

(2). Construct frequency tables classifying all the plastering jobs, for which data were collected, according to:

the rate per hour to plasterers
the area plastered per hour
the cost of plastering per square yard.

Labor Productivity and Cost in Specified Building Trades, by Cities

Plastering

City and job number	Union or nonunion	Rate per hour	Net area plastered per hour (square yds.)	Cost of plastering per square yard
Atlanta, Ga.:				
1	Nonunion	$1.000	2.2	$0.45
2	"	1.000	6.4	.16
3	"	1.000	6.1	.16
4	"	.800	5.2	.15
5	"	.800	5.8	.14
6	"	.800	5.4	.15
7	"	.900	7.7	.12
8	Union	1.100	3.8	.29
9	"	1.100	3.5	.31
10	"	1.100	5.6	.20
11	"	1.100	3.7	.30
12	"	1.250	4.6	.27
13	"	1.000	3.6	.28
14	"	1.000	2.5	.40
15	"	1.000	2.8	.35
16	"	1.250	8.7	.14
17	"	1.250	8.8	.14
18	"	1.250	5.9	.18
19	"	1.000	6.1	.19
Average		$1.040	5.2	$0.23
Birmingham, Ala.:				
1	Nonunion	$0.850	7.1	$0.12
2	"	.850	6.7	.13
3	"	.850	7.2	.12
4	Union	1.000	5.8	.17
5	"	1.000	4.9	.20
6	"	[1]	4.7	.23
7	"	[2]	4.8	.22
Average		$0.957	5.9	$0.17
Chattanooga, Tenn.:				
1	Union	$1.250	4.5	$0.27
2	"	1.100	8.7	.12
3	"	1.100	7.3	.14
4	"	1.100	7.7	.14
5	"	1.100	6.8	.15
6	"	1.000	6.5	.18
7	"	1.250	6.7	.19
8	Nonunion	1.250	5.4	.23
9	"	1.250	4.8	.26
10	Union	1.250	5.9	.21
Average		$1.134	6.4	$0.19

[1] 17 cents per square yard. [2] 22 cents per square yard.

City and job number	Union or nonunion	Rate per hour	Net area plastered per hour (square yds.)	Cost of plastering per square yard
New Orleans, La.:				
1	Union	$1.000	6.5	$0.15
2	"	1.000	6.6	.15
3	"	1.000	3.9	.26
4	"	1.000	3.5	.29
5	"	1.000	4.7	.21
6	"	1.000	2.7	.38
7	"	1.000	3.7	.27
8	"	1.000	4.8	.21
Average		$1.000	4.6	$0.24
Norfolk, Va.:				
1	Union	$1.250	6.6	$0.19
2	"	1.250	7.2	.17
3	Nonunion	[1]	5.8	.28
4	"	[1]	5.9	.27
5	"	1.000	5.6	.18
6	"	1.250	5.4	.23
7	"	1.000	5.3	.19
Average		$1.283	6.0	$0.22
Boston, Mass.:				
1	Union	$1.250	2.7	$0.46
2	"	1.625	4.7	.35
3	"	1.625	4.4	.37
4	"	1.250	4.7	.27
5	"	1.250	4.7	.27
6	"	1.625	6.0	.27
Average		$1.438	4.5	$0.33
New York, N. Y.:				
1	Union	$1.750	2.7	$0.65
2	"	2.000	3.7	.54
3	"	1.875	4.4	.43
4	"	2.000	4.3	.47
5	"	1.875	6.5	.29
6	"	1.875	8.0	.23
7	"	1.875	7.8	.24
8	"	1.875	7.5	.25
9	"	1.750	3.9	.45
Average		$1.875	5.4	$0.39
Philadelphia, Pa.:				
1	Union	$1.750	7.9	$0.22
2	"	1.750	9.0	.19
3	"	1.750	8.8	.20

[1] 22 cents per square yard.

Continued on next page.

City and job number	Union or nonunion	Rate per hour	Net area plastered per hour (square yds.)	Cost of plastering per square yard
4	Nonunion	$1.500	6.5	$0.23
5	"	1.500	6.5	.23
6	"	1.500	7.2	.21
7	Union	1.750	8.7	.20
Average		$1.643	7.8	$0.21
Chicago, Ill.:				
1	Union	$1.875	6.7	$0.28
2	"	1.750	3.6	.49
3	"	1.875	6.5	.29
4	"	1.875	6.7	.28
5	"	1.875	8.0	.23
6	"	1.550	5.6	.28
7	"	1.625	5.9	.27
8	"	1.875	6.6	.28
9	"	1.875	6.3	.30
10	"	1.875	6.9	.27
11	"	1.750	4.6	.38
12	"	1.875	6.9	.27
13	"	1.625	5.6	.29
Average		$1.792	6.1	$0.30
Cincinnati, Ohio:				
1	Nonunion	$1.500	8.4	$0.18
2	"	1.500	7.5	.20
3	"	1.500	7.2	.21
4	Union	1.625	5.5	.30
5	"	1.625	9.4	.17
6	"	1.625	7.3	.22
7	"	1.625	7.8	.21
8	"	1.625	4.4	.37
Average		$1.578	7.2	$0.23
Cleveland, Ohio:				
1	Union	$1.500	6.4	$0.23
2	"	1.500	3.9	.38
3	"	1.500	4.5	.33
4	"	1.500	4.4	.34
5	"	1.500	7.9	.19
6	"	1.500	5.6	.27
7	"	1.500	8.7	.17
8	"	1.500	9.1	.16
9	"	1.500	4.0	.38
10	"	1.500	6.8	.22
Average		$1.500	6.1	$0.27

City and job number	Union or nonunion	Rate per hour	Net area plastered per hour (square yds.)	Cost of plastering per square yard
Denver, Colo.:				
1	Union	$1.500	6.1	$0.24
2	"	1.500	6.7	.22
3	"	1.500	7.5	.20
4	"	1.500	6.9	.22
5	"	1.500	7.2	.21
6	"	1.500	7.1	.21
7	"	1.500	5.2	.29
8	"	1.500	7.5	.20
9	"	1.500	5.5	.27
10	"	1.500	7.2	.21
11	"	1.500	5.9	.36
Average		$1.500	6.6	$0.23
Detroit, Mich.:				
1	Union	$1.563	6.3	$0.25
2	"	1.563	8.6	.18
3	"	1.563	8.8	.18
4	"	1.563	4.0	.39
5	"	1.563	5.6	.28
6	"	1.563	4.6	.34
7	"	1.563	6.4	.24
8	"	1.563	5.4	.29
9	"	1.563	5.5	.29
10	"	1.563	8.5	.18
11	"	1.563	6.7	.23
Average		$1.563	6.4	$0.26
Indianapolis, Ind.:				
1	Nonunion	$1.250	7.9	$0.16
2	"	1.250	7.9	.16
3	Union	1.350	5.9	.23
4	"	1.350	5.1	.27
5	"	1.350	4.5	.30
6	"	1.350	5.2	.26
7	"	1.350	5.2	.26
Average		$1.322	6.0	$0.23
Minneapolis, Minn.:				
1	Union	$1.125	9.9	$0.11
2	"	1.125	8.8	.13
3	"	1.125	9.2	.12
4	"	1.125	6.5	.17
5	"	1.125	6.3	.18
6	"	1.250	7.1	.18
7	"	1.250	6.3	.20
8	"	1.250	9.8	.13
9	"	1.250	9.0	.14
10	"	1.125	3.7	.31
11	"	1.125	4.0	.28
Average		$1.170	7.3	$0.18

3. Assume that you have been asked to investigate the cost of laying bricks in different cities of the United States. Draw up a questionnaire such as you would send to building contractors to elicit the information given in problems C–1 and any other information you consider desirable. Include instructions to informants which will insure the comparability of the figures secured from different individuals.

(It is assumed that these questionnaires are to be sent by mail to the informants.)

4. Draw up a schedule of questions such as might be used in studying the productivity of plasterers and plastering costs in the United States. Detailed information covering all phases of the general problem is to be secured.

In addition to the questionnaire construct a set of general instructions to be followed by each investigator.

(It is assumed that these schedules are to be circulated by hired agents who will visit the informants in person.)

E. Description of the Frequency Distribution

1. The table given below contains a list of the amounts of coal mined in a day by fifty miners in a certain mine in Illinois.

(1). Arrange these items in the form of an array (*Text* p. 63).

(2). Construct a frequency distribution from this list, using a class interval of ~~one-half a ton~~ 5 cents with 3.5 tons as the midpoint of the first class. 15 cents

(3). Construct a histogram based upon this distribution.

(4). Draw a frequency polygon based upon the same distribution.

(5). From the individual items compute the average amount of coal produced per man (i.e., divide the sum of the actual items by the number of cases).

(6). From the frequency distribution calculate the arithmetic average. Do the two averages agree? No Should they?

Why should they agree or disagree?

	Tons of Coal Mined		Tons of Coal Mined
R. Hawkins	3.96	Joe Daniels, Jr.	6.63
Lu Cobb	4.27	Jesse Moore	7.00
Pat Clancy	5.29	S. B. Hopkins	8.49
C. W. Ingram	6.12	P. Deimskrelis	4.98
A. Hoffart	6.56	A. Dascotte	5.32
Joe Calburn, Sr.	4.51	C. Dessent	6.89
W. Miller	6.45	L. A. Badgeley	8.40
L. Dughen	6.91	R. Dascotte	7.57
Leon Clement	9.78	E. Dascotte	7.89
C. Chubert	4.02	Jno. Kegner	6.09
W. C. Roth	6.58	C. Karinski	5.47

Continued on next page.

	Tons of Coal Mined		Tons of Coal Mined
Jules Clement	9.15	E. Eubanks	6.73
Tom Phillips	4.84	Jake Kruski	5.17
B. Clonsting	3.92	Clay Harmon	6.09
Andy Howard	6.06	H. Coleman	6.51
H. Kinsey	7.99	Bert Novhy	4.90
D. Hopi	9.15	N. Albaugh, Sr.	6.72
Tidelk Bassart	7.59	A. Williams	3.50
A. Callura	5.98	W. H. Williams	6.63
T. Callura	6.43	P. Kirk	5.80
Jno. Callura	7.41	J. Chimnati	6.19
Felix Nigro	4.54	Jno. Kemper	7.09
Joe Callura	5.59	M. Gobin	6.66
Tom Chapman	7.84	R. Bazen	7.00
Joe Daniels	8.00	C. Gibbiribin	6.69

2. From the frequency table based upon problem E–1 calculate the arithmetic average by the short method (*Text* p. 118). How does this result agree with the arithmetic average calculated by the long method from the frequency distribution? Upon what principle is the short method based? (Sum of deviations from mean = 0 $\Sigma d' = 0$ How does this principle apply in short method.)

Determine the values of: (1) the median, (2) the first and third quartiles, (3) the 4th decile, (4) the mode.

3. The following frequency distribution is to be studied:

Average Daily Output per Man for Ten Representative Mines in the Fairmont, West Virginia Coal Fields, 1914–1922, in 85 Observations

Range of Output in Tons per Man per Day	Number of Times this Output was Secured
2.50–2.99	1
3.00–3.49	2
3.50–3.99	7
4.00–4.49	5
4.50–4.99	13
5.00–5.49	22
5.50–5.99	12
6.00–6.49	14
6.50–6.99	2
7.00–7.49	3
7.50–7.99	2
8.00–8.49	2
Total	85

Source: *Bulletin No. 361* of the Bureau of Labor Statistics.

(1). From the above figures construct a cumulative frequency table, cumulated upward.
(2). Draw an ogive based on this table.
(3). By graphic methods determine the values of the first and third quartiles and the median.
(4). From the frequency distribution determine the values of the first and third quartiles and the median.
(5). Compute the mean by the short method.
(6). Calculate the geometric mean.

Summarize your numerical results in a table.

4. From the data of problem D–1 construct a frequency distribution based on the number of bricks laid per hour.

For this distribution compute the following measures of variation and skewness: (*Text* Chap. V)

(1). range
(2). arithmetic mean (by the short method)
(3). quartile deviation
(4). mean deviation from the mean
(5). Bowley's measure of skewness

5. Compute the mean deviation from the ungrouped data in problem E–1, using the summation method. (See *Manual* p. 16).

6. For the frequency distribution employed in problem E–4 compute the following measures:

(1). standard deviation
(2). coefficient of variation
(3). Pearson's measure of skewness

7. Two frequency distributions based upon the batting averages of players in the American and National leagues during the early part of the 1925 season are presented below:

Batting Average	Number of players with given average in National League	Number of players with given average in American League
.050–.099	3	0
.100–.149	7	11
.150–.199	11	11
.200–.249	21	22
.250–.299	31	35
.300–.349	34	28
.350–.399	18	13
.400–.449	4	6
.450–.499	0	0
.500–.549	3	2
.550–.599	0	1

Source: *New York Herald-Tribune*, May 17, 1925.

Compare the two leagues with respect to (*a*) average batting ability, (*b*) variation in batting ability. In comparing variability what measures should be employed? Contrast the results secured from different measures of variability.

8. From the following frequency distribution compute all measures of central tendency, variation and skewness necessary for its description.

Adult Women in Ohio Industries in 1922
Classified by Weekly Earnings[1]

Range of week's earnings	Number of women earning specified amounts
$ 0 –$ .99	16
1.00– 1.99	47
2.00– 2.99	72
3.00– 3.99	98
4.00– 4.99	127
5.00– 5.99	133
6.00– 6.99	188
7.00– 7.99	233
8.00– 8.99	265
9.00– 9.99	289
10.00–10.99	417
11.00–11.99	345
12.00–12.99	436
13.00–13.99	335
14.00–14.99	327
15.00–15.99	283
16.00–16.99	153
17.00–17.99	96
18.00–18.99	95
19.00–19.99	68
20.00–20.99	55
21.00–21.99	33
22.00–22.99	29
23.00–23.99	14
24.00–24.99	11
Total	4,165

Source: *Women in Ohio Industries*, U. S. Department of Labor.

9. Analyze and compare the characteristics of the two frequency distributions which follow.

[1] For the purposes of this problem, thirty-one women earning between $25.00 and $35.00 have been excluded from this table.

Pick Miners in Illinois Coal Mines Classified According to Average Daily Earnings (1918–1922)

Range of Average Daily Earnings	Number of Pay Checks In 21 Central Illinois mines	In 52 Southern Illinois mines
$ 2.00– 2.99	501	87
3.00– 3.99	1,288	131
4.00– 4.99	3,222	306
5.00– 5.99	6,293	563
6.00– 6.99	9,821	973
7.00– 7.99	13,089	1,530
8.00– 8.99	11,869	2,684
9.00– 9.99	9,484	5,584
10.00– 10.99	6,748	2,426
11.00– 11.99	4,418	1,433
12.00– 12.99	2,551	853
13.00– 13.99	1,304	577
14.00– 14.99	696	364
15.00– 15.99	362	197
16.00– 16.99	196	105
17.00– 17.99	115	71
18.00– 18.99	57	35
19.00– 19.99	39	33
20.00– 20.99	25	13
21.00– 21.99	16	6
22.00– 22.99	13	7
23.00– 23.99	10	4
24.00– 24.99	10	4
Totals	72,127	17,986

Source: Adapted from *Illinois Miners' Earnings.* Published jointly by three coal operators' associations in Illinois.

Your analysis should include:

(1). Measures of central tendency
- (*a*) arithmetic mean
- (*b*) median
- (*c*) mode

(2). Measures of variation
- (*a*) range
- (*b*) quartile deviation
- (*c*) mean deviation from the mean
- (*d*) standard deviation
- (*e*) coefficient of variation

(3). Measures of skewness
- (*a*) Bowley's measure
- (*b*) Pearson's measure

(4). Graphic comparison
- (*a*) frequency polygons
- (*b*) ogives

Present your numerical results in a table and briefly comment on the comparison.

The following series constitute additional material suitable for substitution in any of the problems of this section.

Meal Checks at the Columbia University Commons for the Week of October 5–11, 1924, Classified According to Amount

Range of amounts spent for each meal cents	Mid-value cents (m)	Number of checks falling in stated classes		
		Breakfast (f)	Lunch (f)	Dinner (f)
2.50– 7.49	5	6	13	13
7.50– 12.49	10	63	47	25
12.50– 17.49	15	116	81	41
17.50– 22.49	20	342	158	55
22.50– 27.49	25	296	274	76
27.50– 32.49	30	209	428	139
32.50– 37.49	35	195	572	270
37.50– 42.49	40	94	613	283
42.50– 47.49	45	57	564	402
47.50– 52.49	50	30	435	418
52.50– 57.49	55	9	261	403
57.50– 62.49	60	4	139	299
62.50– 67.49	65	1	66	204
67.50– 72.49	70	2	31	111
72.50– 77.49	75	0	23	54
77.50– 82.49	80	0	4	29
82.50– 87.49	85	0	1	17
87.50– 92.49	90	0	1	15
92.50– 97.49	95	0	0	7
97.50–102.49	100	0	0	4
102.50–107.49	105	0	0	0
107.50–112.49	110	0	0	2
112.50–117.49	115	0	0	0
117.50–122.49	120	0	0	0
122.50–127.49	125	0	0	0
127.50–132.49	130	0	0	1
Totals		1,424	3,711	2,868

Class A Gas Plants in the United States Classified According to the Prices Charged per Thousand Cubic Feet of Gas in 1918 and 1920

Rate per thousand cubic feet	Number of plants charging stated amounts	
	1918	1920
$ .60– .69	1	0
.70– .79	3	0
.80– .89	3	1
.90– .99	18	2
1.00– 1.09	22	8
1.10– 1.19	20	8
1.20– 1.29	47	9
1.30– 1.39	56	17
1.40– 1.49	72	27
1.50– 1.59	65	53
1.60– 1.69	44	34
1.70– 1.79	24	27
1.80– 1.89	27	20
1.90– 1.99	15	15
2.00– 2.09	8	16
2.10– 2.19	5	15
2.20– 2.29	4	14
2.30– 2.39	3	11
2.40– 2.49	2	8
2.50– 2.59	2	5
2.60– 2.69	0	3
2.70– 2.79	0	2
2.80– 2.89	0	1
2.90– 2.99	0	1
Totals	441	297

Further practice in the description of the frequency distribution may be had from the data presented in problems D–1 and D–2 and in problems given in later pages of this **Manual.** *The following are also suggested as materials that are readily available:*

(1) *Dividend paying preferred stocks classified by price and by dividend rate.*
(2) *All common stocks classified by price.*
(3) *5% bonds classified by price.*
(4) *Foreign exchange rates for the principal European countries, 1914–1925 (similar to the analysis on page 151 of the* **Text**).
(5) *Cash register checks for meals at the local commons.*

In the study of frequency distributions the interest of the work may be enhanced by computing comparable measures for related series, as in Problem 8 and in the alternative distributions given above. Thus preferred and common stocks may be compared with respect to average value, variability and skewness, or sterling (or other foreign exchange) rates before and since 1914 may be similarly compared.

F. Index Numbers of Prices

1. From publications of the Bureau of Labor Statistics it is learned that the average price at wholesale of dressed mutton, per pound, has fluctuated as follows, from 1913 to 1925:

Year	Average price per pound	Year	Average price per pound
1913	$.103	1920	$.162
1914	.101	1921	.104
1915	.107	1922	.120
1916	.125	1923	.119
1917	.166	1924	.145
1918	.198	1925 *June*	.132
1919	.167		

(1). Reduce these to relatives with the price in 1913 = 100.
(2). Change the base to 1919. That is, reduce these prices to relatives with the price in 1919 = 100.

2. The following table lists the relative average price per pound, at wholesale, of live fowls at Chicago from 1913 to 1925.
Change the base of these relatives from 1913 to 1919.

Year	Relative	Year	Relative
1913	100.0	1920	199.8
1914	94.2	1921	162.3
1915	88.6	1922	140.8
1916	106.2	1923	134.8
1917	128.2	1924	141.1
1918	168.3	1925 *June*	160.9
1919	184.2		

3. Using the following quotations, construct index numbers to measure the change in the general level of wholesale grain prices between 1913 and March, 1925.

Grain	Unit	Average Price, 1913	Relative Price, 1913	Average Price, June, 1925	Relative Price, June, 1925
Wheat	bu.	$.913	100	$1 618	177.1
Corn	bu.	.625	100	1.099	175.7
Oats	bu.	.376	100	.505	134.5
Rye	bu.	.636	100	1.089	171.1

Index numbers of the following types are to be constructed:

(1). An aggregate of actual prices.

(2). A relative index number based on aggregates of actual prices (1913 as base).

(3). A simple average of relative prices.

(4). A weighted arithmetic average of relative prices. (Use the following weights: Wheat 14; corn 4; oats 1; rye 1.)

(5). A weighted aggregate of actual prices, reduced to relative terms. Quantities marketed in 1919 are to be used as weights in (5). In round numbers, these were as follows:

Wheat	800	million bushels
Corn	400	" "
Oats	300	" "
Rye	100	" "

4. The difference in the average level of wholesale prices between 1913 and March, 1925, is to be measured. Index numbers are to be constructed from the prices of the commodities listed below. (In order to simplify the calculations only five commodities are quoted. A reliable index should, of course, be based upon a much greater number.)

Commodity	Average, 1913	Average, June, 1925
Wheat, No. 1, northern spring, per bu.	$ .913	$1.815
Coal, bituminous, Norfolk, Va., mine run, Pocahontas, per gross ton	3.000	4.350
Cotton, middling upland, New York, per lb.	.128	.242
Wool, Ohio, fine clothing, per lb.	.589	.450
Leather, harness, oak, California, No. 1, per lb.	.401	.441

Index numbers of the following types are to be constructed:

(1). Unweighted aggregates of actual prices, expressed as relatives with 1913 as base.

(2). Weighted aggregates of actual prices, expressed as relatives using 1913 as base. Prices are to be weighted by the quantities marketed in 1919, which follow:

Commodity	Quantity marketed in 1919
Wheat	807,816,000 bu.
Coal, bituminous	363,125,000 gross tons
Cotton	5,710,382,000 lbs.
Wool	367,128,000 lbs.
Leather, sole	296,326,000 lbs.

(The figures for quantities marketed relate to all grades of the given commodities, not merely to the grades for which prices are given above.

Prices of the grades named are to be taken to represent all grades of the various commodities, and are to be weighted by the total quantities marketed.)

(3). Relatives based upon aggregates of prices per pound. (Assume 60 pounds of wheat to the bushel.)
(4). Medians of relative prices.
(5). Simple arithmetic averages of relative prices.
(6). Weighted arithmetic averages of relative prices.

The weights to be used here are based upon the total values of the various commodities marketed in 1919.

The following table, showing the percentage distribution of these values, gives the weights to be employed:

Wheat	35
Coal, bit	26
Cotton	29
Wool	7
Leather, sole	3
Total	100

(7). Simple geometric averages of relative prices.
(8). Weighted geometric averages of relative prices, using the same weights as in (6)
(9). Simple harmonic averages of relative prices.
(10). Weighted harmonic averages of relative prices, using the same weights as in (6)

Summarize in a single table the ten sets of relatives secured. The report should include tables showing the method employed in each case.

Material suitable for alternative use in this and other problems on index numbers is readily available in ***Wholesale Prices of Commodities****, a pamphlet published monthly by the Bureau of Labor Statistics, and in an annual bulletin, published by the same Bureau in its wholesale price series.*

5. It is desired to measure the changes that have occurred between 1915 and 1920 in the prices of farm implements. The table given below contains a list of such prices. Construct the following types of index numbers from these figures:

(1). Relatives based upon simple aggregates of actual prices, using 1913 as the base year.
(2). A simple average of relative prices.
(3). A geometric average of relative prices.
(4). A harmonic average of relative prices.
(5). A median of relative prices.
(6). A weighted average of relative prices, using the following weights:

Binder	1	Harrow	2
Mower	1	Drill	1
Gang plow	1	Wagon	2
Cultivator	2		

(7). A weighted geometric mean of relative prices.

(8). A weighted harmonic mean of relative prices.

(9). A weighted median of relative prices.

Wholesale Prices of Certain Representative Farm Implements

Implements	1915	1916	1917	1918	1919	1920
Binder, 7 ft. cut	$117.50	$118.50	$153.45	$198.50	$198.50	$187.50
Mower, 5 ft. cut	38.00	38.00	49.45	66.00	66.00	63.50
Gang plow, 14-inch	55.50	61.60	75.90	111.00	111.50	111.00
Cultivator, 2-horse rider	24.75	26.40	34.20	51.00	51.25	51.00
Harrow, 20 ft.	25.00	27.50	26.40	38.00	43.50	40.00
Drill, 10 ft.	92.00	93.45	112.20	152.00	152.75	152.75
Wagon, 3¼x10	73.50	77.15	86.90	123.75	138.50	138.50
Total	$426.25	$442.60	$538.50	$740.25	$762.00	$744.25

Source: *Wages of Farm Labor*, a bulletin of the Agricultural Experiment Station of the University of Minnesota.

6. A comparative study is to be made of any two of the following American index numbers, from 1919 to the present time, by months:

Bureau of Labor Statistics index number of wholesale prices.

Bradstreet's index number of wholesale prices.

Persons' ten commodity index number of business cycles.

The American twenty basic commodity index number of the Federal Reserve Bank of New York.

The index number of the Federal Reserve Board.

(1). For what purpose is each index number constructed?

(2). Explain in detail the methods employed in constructing these indexes.

(3). What commodity prices are employed? What weights are used?

(4). Which index is the more sensitive to business conditions?

The two index numbers should be plotted, on a comparable basis, for the period in question. All important differences in the time and magnitude of their movements should be noted and explained, if possible.

7. A comparison is to be made of monthly changes in the general level of prices in the United States and in either England, France, Canada, or Japan, since 1919.

Construct a chart from the Federal Reserve Board's index numbers compiled for the purpose of making international price comparisons. Analyze the curves with respect to rates of increase and decrease, relative price levels, times of highest and lowest prices. The comparison should be made on a gold basis and on a currency basis.

Reference: *Prices in the United States and Abroad*, Federal Reserve Board.

8. Study the monthly price changes from 1919 through 1925 that are revealed in the U. S. Bureau of Labor Statistics index number of wholesale prices. Compare this index with one of the following:

(*a*) The Statist index of wholesale prices for England.

(*b*) Dominion Bureau of Statistics index of wholesale prices for Canada.

(*c*) Bureau de la Statistique Générale index of wholesale prices for France.

(*d*) Ricardo Bachi's index of wholesale prices for Italy.

(*e*) Central Bureau of Statistics index of wholesale prices for the Netherlands.

9. Compare graphically on a single chart the Bureau of Labor Statistics monthly price index numbers for major commodity groups, since January, 1919. In a brief note present a summary of your conclusions.

10. Construct a chart showing monthly fluctuations since January, 1919, in the price index numbers for the different groups into which the Federal Reserve Board divides the Bureau of Labor Statistics commodities. Account for the differences. Discuss.

11. The following table shows the annual prices at wholesale of six mineral products, from 1919 to 1924.

Product	Unit	Average Wholesale Price					
		1919	1920	1921	1922	1923	1924
Pig iron	Gross ton	$28.970	$42.760	$22.580	$24.060	$26.290	$20.910
Copper	Pound	.189	.175	.127	.136	.146	.132
Coal, Anthracite	Gross ton	8.270	9.500	10.530	10.980	11.760	11.430
Coal, Bituminous	Net ton	2.340	5.930	2.190	2.790	2.270	1.930
Coke	Net ton	4.580	11.300	3.450	7.080	5.340	3.420
Petroleum	Per bbl.	2.280	3.400	1.700	1.810	1.440	1.450

The amounts of these minerals produced annually during this period are given in Section IV.

Construct an index number of each of the following types to measure the changes in mineral prices during this period:

(1). Aggregates of actual prices weighted by base year quantities. (*Text* p. 207.)

(2). Aggregates of actual prices weighted by given year quantities. (*Text* p. 207.)

(3). "Ideal" index; Geometric mean of the two preceding index numbers. (*Text* p. 215, formula 48 in *Manual.*)

(4). "Best practical all round formula." (*Text* p. 218, formula 49 in *Manual.*)

(5). A chain index with 1919 as base, based upon link "Ideal" index numbers computed for the several years. (*Text*, footnote p. 218).

G. Analysis and Combination of Time Series

1. The figures showing the annual production of coke in the United States since 1890 will be found in Section IV of this *Manual*. It is desired to measure the secular trend for this series.

(1). Calculate a five year moving average of these figures.
(2). Calculate a nine year moving average of these figures.
(3). Construct a chart showing the actual data and the moving averages.
(4). Measure the deviations of the actual data from each of these moving averages. Compare these sets of deviations and explain their differences.
(5). What do these deviations represent?

2. Fit a straight line by the method of least squares to the following seven points:

x	y
1	1
2	2
3	2
4	3
5	4
6	4
7	5

Plot the points and the line on arithmetic coördinate paper. Compare the observed values of y with those computed from the equation to the fitted line.

Measure the deviations (d). Compute the values of $\Sigma(d)$ and $\Sigma(xd)$. (*Text* p. 278.)

3. Fit a straight line by the method of least squares to the following seven points:

x	y
− 3	1
− 2	2
− 1	2
0	3
1	4
2	4
3	5

Plot the points and the line on arithmetic coördinate paper. Compare the observed values of y with those computed from the equation to the fitted line.

Measure the deviations (d). Compute the values of $\Sigma(d)$ and $\Sigma(xd)$.

Compare the results secured in this exercise with those secured in problem 2 above.

4. On ordinary arithmetic cross section paper chart the series representing the importation of crude rubber [1] into the United States from 1914 to 1919 inclusive. By inspection decide upon the location of a straight line that to your eye best describes the trend in rubber imports. The use of a transparent celluloid triangle will facilitate this operation.

Then calculate the equation to the straight line which by the method of least squares is the best fit.

How do the two lines compare? Is the second line a better fit than the first? What was your criterion of best fit in the first case? Would an extension of either line afford a logical basis for a forecast of the importation of rubber for the next year? For the next ten years? Explain your answer.

Extend the trend in a broken line up to date, and plot in the actual figures from 1919 to the present time.

How do the forecasts you would have made in 1919 compare with what actually happened? How do you explain the departures from the trend?

5. Determine the equation to a straight line of trend for coke production in the U. S. from 1890 to date. (Section IV.)

Plot this line of trend on the chart prepared in working on Problem G–1.

Measure the deviations of the actual values from the straight line trend values.

Compare the deviations thus secured with the deviations secured from the moving averages determined in problem G–1.

Of the three lines of trend, which seems the best? Why?

6. The following table gives the total amount of new capital issues in the U. S. from 1914 to 1924:

	New capital issues (in billions of dollars)
1914	1.4
1915	1.4
1916	2.2
1917	1.5
1918	1.3
1919	3.0
1920	3.1
1921	2.6
1922	3.4
1923	3.6
1924	3.2

[1] This problem may be varied by substituting such series as the production of corn, wheat, cotton, pig iron, cement, etc., which will be found in Section IV.

Determine the equation to a straight line of trend fitted to these figures.

Determine the trend value for each year.

Determine the trend value for each month of 1923.

7. Construct a rectilinear chart of coke production in the United States, plotting annual production figures for the period 1904–1924 inclusive.

Fit a line of trend of the type

$$y = a + bx + cx^2$$

Plot the curve secured. Measure the deviations from this curve.

Construct a new chart showing graphically the deviations from the second degree parabola and from the straight line fitted to the data of coke production (Problem G–5). Make a detailed comparison of these two sets of deviations. Account for the differences.

Does the parabola or the straight line afford a better measure of trend within the period 1904 to 1924? Which affords the more logical basis for forecasting coke production?

8. Construct a chart on semi-logarithmic paper of coke production in the United States from 1895 to date.

(1). Fit to this series a line of trend of the type $\log y = a + bx + cx^2$.

(2). Compute the trend value for each year of this period and plot the line of trend on the chart.

(3). Express the actual values as percentages of the trend values.

(4). Present in tabular form the actual annual figures, the trend values and the percentages computed as indicated.

(5). Explain the deviations in the light of business conditions prevailing during the period covered.

9. It is desired to measure the seasonal variation in cement production in the United States. Section IV gives the monthly production of cement from 1913 to 1924.

Index numbers of seasonal variations are to be computed, employing the ratio to trend method.

(1). Fit a straight line to the annual data (1913–1924).

(2). Compute the monthly trend values.

(3). Express the actual monthly values as percentages of the trend values.

(4). Construct a multiple frequency table of these figures. (*Text* p. 325.)

(5). Bearing in mind the considerations mentioned in the *Text*, construct index numbers of seasonal variation by averaging a selected number of central items in each of the monthly frequency distributions. Adjust these so that their average is 100. (*Text* p. 326.)

10. Compute index numbers of seasonal variation for the same data from ratios to the trend secured by fitting a second degree parabola.

11. Compute index numbers of seasonal variation for the same data from ratios to a twelve months moving average. (*Text* p. 321.)

12. Compute index numbers of seasonal variation for the same data by averaging the actual values and making the necessary correction for linear trend. (*Text* p. 316.)

13. Compute index numbers of seasonal variation for the same data by the method of link relatives. (*Text* p. 318.)

Note: Any of the monthly series given in section IV may be substituted for cement production. If desired, different series may be assigned to different members of the class.

14. It is desired to study the cyclical fluctuations in cement production, using monthly data.

(All the steps usually taken in analyzing a time series for the purpose of studying cyclical movements are outlined below, although certain of these have been completed for the data of cement production.)

(1). Plot the annual cement production figures from 1913 to 1924 and fit an appropriate line of trend. (Section IV.)

(2). Compute the monthly trend values for the entire period. (If the trend is non-linear the method of interpolating for monthly values suggested in the *Text* (pp. 334–336) may be employed.)

(3). Express the actual monthly values as percentages of the corresponding trend values.

(4). Calculate the index numbers of seasonal variation, employing the ratio to trend method (or any other appropriate method).

(5). Correct the percentage values calculated in (3) by the index numbers of seasonal variation. The corrected values measure the effect of accidental and cyclical deviations from trend. If necessary they may be reduced to terms of the root-mean-square (standard) deviation, as explained in the next step.

(6). Compute the root-mean-square of the percentage deviations. Divide each of the monthly variations from trend by this value.

(7). Plot the data secured from step (5) [or step (6)] about a heavy horizontal line representing normal.

(See *Text* p. 337 for a suggested form which may be employed in performing the operations outlined above.)

15. Construct annual index numbers of the physical volume of mineral production from 1919 to 1924, using the six series which were employed in problem F–11. Employ for this purpose the "Ideal" index as applied to quantities (formula 50 of this *Manual*).

16. It is desired to construct index numbers measuring the cyclical fluctuations in mineral production from 1904 to 1924, using annual data.

(1). Fit an appropriate line of trend to each of the six mineral series [1] employed in G–15. (Annual data for the period in question are given in section IV.)

(2). For each series measure the deviations of actual from trend values. Compute the root-mean-square of these values and express the individual deviations in terms of the root-mean-square (standard) deviation.

(3). Compute weights measuring the relative importance of these minerals, basing these calculations upon the values of the amounts produced in 1924. (See problem G–15.) Later calculations will be facilitated if these weights are rounded off to total 100.

(4). Calculate for each year a weighted arithmetic average of the deviations of the six series, as computed in (2) above. These averages constitute the desired index numbers.

(5). Plot these figures about a heavy horizontal line representing normal.

Note: Index numbers of physical volume may be computed on a monthly basis by combining, in the form of a weighted average, data for a number of series similar to the figures secured for cement production in step (6) problem G–14 above.

The other methods of constructing such physical volume index numbers which are illustrated in the ***Text*** *may be employed, using appropriate data.*

H. Correlation; the Measurement of Relationship Between Two Variables

1. The following is a record of eleven farms, showing the number of bushels of wheat secured per acre and the calculated cost per bushel:

Farm number	Yield per acre	Cost per bushel
1	9.13 bu.	$1.84
2	11.37	1.36
3	8.00	2.00
4	9.80	1.56
5	10.81	1.68
6	18.12	.98
7	18.00	.53
8	20.22	.67
9	15.59	.72
10	8.72	2.04
11	14.00	.93

[1] For class purposes it may be desirable to assign specific series to individual students for analysis, the final work of combination being performed by all the members of the class.

Represent the relation between these two factors graphically by plotting a point for each farm. Let yield per acre be represented on the x axis and cost per bushel on the y axis. This chart is known as a "scatter diagram."

Determine the equation to the line that best describes the average relationship. Plot the line on the scatter diagram. From the equation to the line of average relationship calculate the theoretical cost per bushel for each farm. Do the actual costs vary from the theoretical costs? How may we measure the amount of this variation?

2. It is desired to measure the degree of correlation that exists between corn yield and land values in Iowa. The table below presents data bearing on this subject for twenty-five counties of Iowa.

County	Average land value per acre Jan. 1, 1920 X_1	Average corn yield per acre, in bushels 1910–1919 X_2
Allamakee	$ 87	40
Bremer	133	36
Butler	174	34
Calhoun	285	41
Carroll	263	39
Cherokee	274	42
Dallas	235	40
Davis	104	31
Fayette	141	36
Fremont	208	34
Howard	115	30
Ida	271	40
Jefferson	163	37
Johnson	193	41
Kossuth	203	38
Lyon	279	38
Madison	179	34
Marshall	244	45
Monona	165	34
Pocahontas	257	40
Polk	252	41
Story	280	42
Wapello	167	35
Warren	168	33
Winneshiek	115	36

Source: *Correlation and Machine Calculation* by H. A. Wallace and George W. Snedecor of the Iowa State College of Agriculture and Mechanic Arts.

(1). Construct a scatter diagram, using corn yield per acre as the independent variable.

(2). By the method of least squares determine the equation to the line of average relationship between corn yield and land values in these counties.

(3). Calculate the theoretical land value per acre that you would expect to find in each county.

(4). Measure the difference between the actual land value and the computed land value in each case.

(5). Calculate the standard error of estimate, i.e., the root-mean-square of these differences. Compare this with the standard deviation of land values.

(6). Calculate the coefficient of correlation from the formula,

$$r^2 = 1 - \frac{S_y^2}{\sigma_y^2}$$

(7). Summarize your results, explaining the precise significance of each measure.

3. Using the data of the above problem, calculate the standard error of estimate and the coefficient of correlation by the least squares method. (*Text* p. 401.)

4. The figures which follow show, for the year 1920, the number of Ford, Buick and Cadillac automobiles registered in the different states, and the number of persons in each state reporting incomes under $2000, between $2000 and $5000, and over $5000.

Determine the degree of correlation[1] between one of the following combinations:

(1). Ford registration and the lowest income group.

(2). Buick registration and the medium income group.

(3). Cadillac registration and the highest income group.

Is income a good index of potential demand for these cars? How good?

How do you account for the states that fall above and below the lines of regression? Have they any common characteristic? Are the same states above in all three cases?

Summarize your results in a table and indicate the precise meaning of each measure.

This problem may be worked out by either the product-moment or the least squares method. It is desirable that practice be given in both procedures.

[1] Because of the predominant influence of a case as large as New York it would be advisable to exclude it in measuring the correlation. (See *Text*, p. 405.)

Automobile Registration in 1920, by States
(in thousands)

State	Ford	Buick	Cadillac
Alabama	29.6	4.7	.8
Arizona	11.3	1.8	.4
Arkansas	27.2	2.6	.3
California	144.5	33.2	9.4
Colorado	57.2	7.0	1.2
Connecticut	27.8	9.5	3.5
Delaware	7.4	1.3	.3
Florida	26.1	6.0	1.5
Georgia	63.2	9.9	1.5
Idaho	18.6	3.8	.4
Illinois	200.1	30.3	6.9
Indiana	141.5	18.9	2.2
Iowa	185.8	29.7	4.2
Kansas	119.8	19.6	1.3
Kentucky	51.5	6.1	.9
Louisiana	23.3	2.6	.8
Maine	21.2	4.2	1.3
Maryland	34.2	6.0	1.1
Massachusetts	80.4	21.8	9.1
Michigan	151.4	22.6	4.9
Minnesota	138.4	18.8	2.5
Mississippi	29.3	2.8	.4
Missouri	126.8	14.1	2.8
Montana	26.3	3.9	.5
Nebraska	107.0	16.4	1.2
Nevada	3.7	.7	.1
New Hampshire	12.5	2.6	.8
New Jersey	70.9	15.5	6.5
New Mexico	10.9	1.8	.2
New York	178.4	39.4	20.1
North Carolina	60.2	6.4	1.0
North Dakota	47.0	4.6	.4
Ohio	204.4	37.9	7.9
Oklahoma	104.0	11.4	1.2
Oregon	37.2	6.3	1.1
Pennsylvania	201.4	34.4	8.6
Rhode Island	14.0	2.3	1.8
South Carolina	39.0	4.9	.8
South Dakota	54.5	7.3	.3
Tennessee	46.8	5.3	.8
Texas	196.3	25.0	4.0

Continued on next page

State	Ford	Buick	Cadillac
Utah	14.4	3.0	.4
Vermont	12.0	2.7	1.0
Virginia	54.3	5.4	1.3
Washington	57.1	9.2	1.6
West Virginia	25.5	4.2	.5
Wisconsin	133.2	18.4	1.8
Wyoming	10.3	1.9	.3

Number of Personal Income Tax Returns for 1920, *by States Classified by Size of Income Reported*

(*in thousands*)

State	Under $2000	$2000 to $5000	Over $5000
Alabama	18.1	30.8	4.1
Arizona	8.1	15.3	1.4
Arkansas	9.0	25.9	3.3
California	140.0	211.3	45.7
Colorado	27.2	41.2	5.7
Connecticut	63.3	73.6	11.3
Delaware	8.1	9.4	2.4
Florida	13.6	24.1	5.5
Georgia	23.3	43.8	6.2
Idaho	9.3	15.1	1.3
Illinois	188.8	293.2	60.5
Indiana	64.4	112.4	12.9
Iowa	51.0	107.7	24.7
Kansas	35.4	55.0	8.8
Kentucky	29.2	41.5	7.5
Louisiana	22.9	38.8	7.6
Maine	18.7	25.2	3.8
Maryland	56.7	77.9	13.3
Massachusetts	142.7	224.2	34.8
Michigan	132.3	150.9	21.9
Minnesota	62.0	79.7	12.3
Mississippi	10.9	15.8	2.2
Missouri	60.8	144.2	18.0
Montana	20.1	23.8	1.6
Nebraska	35.0	51.4	11.3
Nevada	4.5	5.5	.4
New Hampshire	15.9	17.8	2.3
New Jersey	108.1	161.1	27.8
New Mexico	4.4	8.5	.8
New York	370.5	551.3	126.0
North Carolina	16.6	26.0	4.7

Continued on next page.

State	Under $2000	$2000 to $5000	Over $5000
North Dakota	8.6	14.3	1.4
Ohio	168.4	242.6	37.0
Oklahoma	23.3	48.2	10.3
Oregon	28.9	33.8	4.9
Pennsylvania	263.2	351.4	58.1
Rhode Island	24.4	24.3	4.5
South Carolina	7.9	21.9	3.2
South Dakota	12.3	19.3	3.0
Tennessee	23.2	34.7	7.3
Texas	66.7	138.3	19.6
Utah	10.4	18.4	1.7
Vermont	8.2	9.4	1.6
Virginia	35.9	49.5	7.2
Washington	72.0	68.4	7.8
West Virginia	31.5	58.6	6.3
Wisconsin	67.7	70.5	12.2
Wyoming	8.4	15.2	.9

Source: *Statistics of Income,* United States Treasury

5. Determine the relationship between the number of grocers and the number of druggists [1] in counties in New York, Pennsylvania and Ohio,[2] as given below.

(1). Construct a correlation table (*Text* p. 391) and tabulate the paired items for each county. To secure uniformity in the work of all class members, it is suggested that the number of druggists be taken as the x variable. A class interval of 5 may be taken in tabulating druggists and an interval of 20 in tabulating grocers.

(2). Calculate the coefficient of correlation by the product-moment method.

(3). Determine the equation to the line of regression of y on x (i.e., y as the dependent variable) and compute the standard error of estimate.

(4). Construct a scatter diagram, plot the line of regression and indicate the limits of the zones of estimate.

(5). Given a county in which there are 50 druggists, how many grocers would you expect to find in the same county? Can you determine the reliability of the estimate?

(6). Determine the equation to the line of regression of x on y (i.e., x as the dependent variable).

[1] Source: *Sales Quotas by Counties,* Curtis Publishing Co.

[2] Nine densely populated counties have been omitted from this list, in order to secure a homogeneous group.

(7). Given a county in which there are 170 grocers, estimate the number of druggists in the same county. Give a measure of the reliability of this estimate.

Number of Grocers and Druggists in New York State Counties in 1920

County	No. of grocers	No. of druggists
Monroe	178	101
Westchester	303	90
Onondaga	199	81
Albany	163	54
Oneida	110	43
Rensselaer	110	39
Orange	121	51
Broome	114	35
Chautauqua	113	34
Nassau	133	32
Niagara	157	30
Steuben	114	31
Schenectady	54	31
St. Lawrence	87	30
Jefferson	122	38
Suffolk	95	42
Dutchess	105	34
Ulster	88	21
Richmond	71	29
Oswego	77	23
Cattaraugus	94	32
Chemung	115	24
Cayuga	65	20
Herkimer	60	18
Saratoga	75	20
Otsego	51	20
Ontario	47	16
Wayne	56	21
Montgomery	68	22
Delaware	47	15
Fulton	45	17
Washington	43	19
Madison	60	17
Alleghany	46	20
Chenango	48	18
Tompkins	38	16
Clinton	42	11
Columbia	37	16

Continued on next page.

County	No. of grocers	No. of druggists
Franklin	53	19
Rockland	46	17
Genesee	41	9
Livingston	42	14
Cortland	40	13
Warren	44	17
Wyoming	34	11
Sullivan	41	18
Essex	31	21
Tioga	36	15
Greene	31	15
Orleans	21	12
Schoharie	24	8
Lewis	15	8
Seneca	18	9
Yates	16	6
Schuyler	16	4
Putnam	12	1
Hamilton	1	0

Number of Grocers and Druggists in Pennsylvania Counties in 1920

Luzerne	238	108
Lackawanna	239	79
Westmoreland	429	80
Berks	160	43
Lancaster	127	48
Schuylkill	259	63
Montgomery	152	73
York	139	37
Dauphin	149	55
Erie	141	39
Lehigh	86	8
Cambria	234	53
Washington	294	43
Fayette	288	40
Northampton	85	40
Delaware	102	63
Blair	140	43
Northumberland	131	44
Chester	94	39
Lycoming	92	34
Beaver	210	40
Clearfield	112	22

Continued on next page.

County	No. of grocers	No. of druggists
Bucks	56	25
Mercer	121	28
Butler	56	26
Lawrence	75	24
Crawford	64	25
Lebanon	49	42
Armstrong	96	25
Cumberland	67	20
Franklin	74	9
Somerset	78	20
Indiana	115	36
Bradford	47	16
Venango	44	17
Jefferson	88	18
Columbia	40	13
Carbon	17	10
McKean	65	23
Centre	38	19
Tioga	29	15
Warren	16	16
Bedford	29	10
Huntingdon	32	7
Adams	15	12
Clarion	36	14
Susquehanna	25	11
Clinton	29	11
Mifflin	17	8
Greene	24	10
Wayne	18	9
Elk	33	11
Perry	22	5
Monroe	15	7
Potter	18	9
Snyder	7	8
Union	12	8
Wyoming	5	9
Juniata	8	5
Montour	13	6
Fulton	5	2
Sullivan	7	3
Pike	5	1
Forest	3	3
Cameron	4	1

Continued on next page.

Number of Grocers and Druggists in Ohio Counties in 1920

County	No. of grocers	No. of druggists
Franklin	200	100
Lucas	171	86
Summit	150	53
Montgomery	150	63
Stark	177	44
Mahoning	110	37
Butler	126	29
Columbiana	154	74
Clark	58	21
Belmont	146	23
Lorain	63	24
Trumbull	94	20
Allen	79	33
Licking	70	22
Muskingum	66	25
Astabula	59	23
Tuscarawas	77	18
Scioto	82	22
Jefferson	127	26
Richmond	51	19
Miami	65	19
Seneca	41	15
Darke	21	14
Athens	40	13
Wood	38	65
Washington	53	14
Marion	68	15
Wayne	42	14
Hancock	33	9
Fairfield	46	21
Guernsey	52	12
Ross	42	16
Sandusky	34	12
Erie	45	17
Crawford	33	16
Portage	30	8
Lawrence	29	8
Huron	30	11
Logan	34	16
Perry	38	12
Knox	25	13
Hardin	15	17

Continued on next page.

County	No. of grocers	No. of druggists
Coshocton	19	9
Clermont	34	13
Highland	24	7
Auglaize	21	11
Greene	28	10
Van Wert	24	11
Ashland	19	12
Williams	26	16
Delaware	32	6
Champaign	20	10
Warren	35	9
Meigs	26	6
Medina	11	7
Putnam	15	14
Preble	21	10
Shelby	17	9
Jackson	26	8
Dickaway	24	10
Defiance	15	11
Clinton	22	10
Fulton	21	10
Mercer	22	10
Lake	32	10
Brown	21	8
Union	14	6
Henry	18	11
Adams	16	6
Fayette	20	6
Hocking	21	5
Wyandot	19	10
Gallia	16	5
Madison	13	7
Monroe	12	6
Ottawa	18	6
Harrison	23	9
Morrow	9	4
Paulding	14	8
Holmes	11	4
Noble	7	2
Carroll	7	4
Morgan	8	5
Geauga	12	5
Pike	6	5
Vinton	3	2

Relation Between Farm Expenses and Farm Receipts (199 Farms Operated by White Owners) — Sumter County, Ga., 1913

Farm expenses (class limits)	Number of farms	Number of farms with specified farm receipts								
		$950 to $1,949	$1,950 to $2,949	$2,950 to $3,949	$3,950 to $4,949	$4,950 to $5,949	$5,950 to $6,949	$6,950 to $7,949	$7,950 to $8,949	$8,950 to $9,949
$450 to $949	25	24	1	.....	.....	.....	.....	.....	.....	.....
$950 to $1,449	39	26	13	.....	.....	.....	.....	.....	.....	.....
$1,450 to $1,949	36	2	20	14	.....	.....	.....	.....	.....	.....
$1,950 to $2,449	30	.....	7	20	3	.....	.....	.....	.....	.....
$2,450 to $2,949	15	.....	2	5	7	1	.....	.....	.....	.....
$2,950 to $3,449	15	.....	.....	1	6	6	.....	1	1	.....
$3,450 to $3,949	7	.....	.....	.....	2	4	.....	.....	1	.....
$3,950 to $4,449	5	.....	.....	.....	.....	4	1	.....	.....	.....
$4,450 to $4,949	7	.....	.....	.....	.....	.....	1	5	1	.....
$4,950 to $5,449	10	.....	.....	.....	.....	2	2	2	2	2
$5,450 to $5,949	6	.....	.....	.....	.....	1	1	1	2	1
$5,950 to $6,449	2	.....	.....	.....	.....	.....	.....	.....	1	1
$6,450 to $6,949	2	.....	.....	.....	.....	.....	.....	.....	.....	2
All	199	52	43	40	18	18	5	9	8	6

Source: U. S. Department of Agriculture, *Department Circular 307*. Because of inequalities in the original class intervals and in order to simplify the present exercise, 38 cases below and 1 case above the given class limits have been omitted.

6. For the cases given in the correlation table on page 90, measure the relationship between farm receipts and farm expenses. The product-moment method (*Text* p. 390) is to be applied.

(1). Calculate the two standard deviations in terms of class-interval units.
(2). Working from the assumed means of both variables, calculate the mean product in terms of class-interval units.
(3). Compute the coefficient of correlation.
(4). Reduce the standard deviations to original units and determine the equations to the two lines of regression, the origin being at the point of averages.
(5). Determine the equations to the two lines of regression, the origin being at the zero point on the original scales.
(6). Compute the two standard errors of estimate.
(7). Summarize and explain your results.

7. Annual figures for the production of pig iron and coke are given in Section IV.

(1). Measure the correlation between deviations from lines of trend fitted to these two series for the period 1895 to date. (If lines of trend were fitted to these figures in connection with the study of time series, the results may be utilized here. It is not essential that the period covered be that suggested above.) All the measures necessary for a complete description of this relationship should be computed.

If the production of pig iron should be 33 million tons during the current year, what would be the most probable figure for the production of coke?

(2). Measure the correlation between year to year variations in the production of pig iron and coke. As in (1) above, compute all necessary measures.

Compare the results secured by these two methods. Account for the differences.

On the assumption of a 33 million ton production of pig iron during the current year, what would be the most probable figure for the production of coke? Compare this forecast with the one made in (1) above and account for the difference.

8. The table on page 92 presents the rates on prime commercial paper in New York, adjusted for seasonal variation,[1] and an index of pig iron production adjusted for seasonal variation and secular trend, for the period 1906–1914.

[1] As there was no upward or downward trend of interest rates no adjustment for trend was necessary.

(1). Present both of these series graphically.

(2). Determine the degree of correlation between these two series when concurrent items are paired.

(3). By inspecting the data determine the approximate degree of lag between the two series, and compute r when the items are paired with this lag. Compute r with a lag less than this tentative figure by two months; greater by two months. Continue until the maximum value of r is secured.

(4). Compute the two equations of regression and calculate the values of S_x and S_y. Compare these latter values with the two standard deviations.

(In these calculations let the index of pig iron production be the x variable and interest rates the y variable.)

Rates on Prime Commercial Paper in New York, Adjusted for Seasonal Variation

	1906	1907	1908	1909	1910	1911	1912	1913	1914	
Jan.	5.20	6.30	6.43	3.87	4.90	4.13	4.05	4.81	4.68	Jan.
Feb.	5.41	6.32	5.44	3.91	4.82	4.47	4.13	5.29	4.22	Feb.
Mar.	5.48	6.39	5.83	3.70	4.70	4.08	4.39	5.95	4.02	Mar.
Apr.	5.71	6.19	4.65	3.77	5.02	3.93	4.42	5.80	4.02	Apr.
May	5.73	5.84	4.34	3.84	5.15	4.03	4.59	5.74	4.28	May
June	5.79	5.98	4.19	3.79	5.35	4.23	4.54	6.42	4.42	June
July	5.71	5.98	3.98	3.61	5.61	4.01	4.76	6.29	4.73	July
Aug.	5.84	6.09	3.43	3.86	5.27	4.03	4.84	5.84	6.18	Aug.
Sept.	6.04	6.29	3.40	3.70	5.01	4.01	5.04	5.26	6.18	Sept.
Oct.	5.71	6.47	3.47	4.44	4.97	3.76	5.33	5.10	5.85	Oct.
Nov.	5.82	7.07	3.60	4.65	5.07	3.48	5.29	5.13	5.07	Nov.
Dec.	5.75	7.50	3.35	4.59	4.16	4.13	5.50	5.18	3.85	Dec.

Source: *The Review of Economic Statistics*, Jan., 1925.

Adjusted Index of Pig Iron Production
(Normal = 100)

	1906	1907	1908	1909	1910	1911	1912	1913	1914	
Jan.	123	124	56	92	126	82	92	119	78	Jan.
Feb.	117	119	63	92	120	88	98	115	82	Feb.
Mar.	120	117	58	86	119	94	99	110	89	Mar.
Apr.	118	119	57	84	115	91	101	112	89	Apr.
May	117	121	56	89	109	82	105	114	80	May
June	116	124	59	97	109	83	108	111	79	June
July	118	126	66	107	104	84	107	109	81	July
Aug.	111	123	71	112	100	88	109	106	86	Aug.
Sept.	113	119	74	118	97	90	107	105	82	Sept.
Oct.	119	120	75	122	93	89	110	100	72	Oct.
Nov.	124	98	81	124	89	89	113	92	65	Nov.
Dec.	125	66	88	127	82	90	118	81	64	Dec.

Source: *The Review of Economic Statistics*, Jan., 1925.

9. From a census of farms in North Dakota in 1922 the facts given below were determined, regarding the yield of wheat, in bushels per acre, and the total cost per bushel of raising wheat.

You are required to determine the nature of the relationship between the cost per bushel and the yield per acre.

(1). Construct a scatter diagram using cost per bushel as the dependent variable.

(2). Fit a line by the method of least squares to these data. Plot this line on the scatter diagram.

(3). Estimate the cost per bushel for each farm from the equation to the least squares line.

(4). Calculate the deviations of the actual costs from the estimated figures.

(5). Compute the standard error of estimate, S_y, and lay off the zones of estimate about the line of average relationship.

(6). Determine the standard deviation of cost per bushel.

(7). Determine the value of the coefficient of correlation.

(8). Interpret your results.

Record of 73 North Dakota Farms for 1922

Farm number	Acres in wheat	Yield per acre (bushels)	Total cost per bushel of wheat
1	15	9.13	$1.84
2	105	11.37	1.36
3	40	8.00	2.00
4	65	9.80	1.56
5	111	10.81	1.68
6	102	8.14	1.07
7	240	10.92	1.29
8	no wheat	no wheat	no wheat
9	113	18.12	.98
10	100	18.00	.53
11	268	20.22	.67
12	85	22.35	.75
13	125	33.60	.64
14	110	30.91	.67
15	135	26.00	.97
16	130	9.98	1.16
17	130	9.62	1.36
18	65	6.62	1.60
19	140	14.00	.93
20	160	13.06	1.26
21	276	18.26	.75

Continued on next page.

Record of 73 North Dakota Farms for 1922 — Continued

Farm number	Acres in wheat	Yield per acre (bushels)	Total cost per bushel of wheat
22	235	10.13	$1.24
23	435	10.57	1.14
24	98	15.31	.98
25	200	19.48	.82
26	102	10.02	1.40
27	270	13.36	.84
28	170	15.59	.72
29	78	24.36	1.04
30	280	21.36	.73
31	195	8.72	2.04
32	405	15.49	.86
33	190	21.05	.76
34	216	11.80	1.08
35	70	15.71	.75
36	205	17.07	.76
37	150	16.00	.73
38	145	13.49	.89
39	75	10.00	1.10
40	170	18.95	.67
41	76	15.47	.94
42	84	20.24	.59
43	no wheat	no wheat	no wheat
44	76	16.61	.86
45	80	17.50	.87
46	160	13.00	.83
47	175	11.43	.92
48	200	15.64	.61
49	420	9.54	1.27
50	205	16.10	.78
51	250	15.75	.59
52	180	24.44	.50
53	147	20.41	.52
54	200	17.00	.61
55	100	15.00	.62
56	160	21.89	.54
57	244.5	20.65	.62
58	235	21.91	.72
59	29	17.38	.88
60	195	22.68	.72
61	345	20.96	.48

Continued on next page.

Record of 73 North Dakota Farms for 1922 — Continued

Farm number	Acres in wheat	Yield per acre (bushels)	Total cost per bushel of wheat
62	96	26.00	$0.48
63	206	27.67	.49
64	84	14.29	.84
65	54	24.56	.58
66	90	10.00	.88
67	290	20.81	.37
68	75	11.33	.99
69	190	14.00	.60
70	110	19.00	.51
71	134	13.59	.85
72	179.75	9.38	1.01
73	117	11.50	.76

Data supplied through the courtesy of the Agricultural Experiment Station, North Dakota Agricultural College.

10. (1). To the data of Problem 9 above fit a second degree parabola by the method of least squares. Plot this line on the scatter diagram constructed for Problem 9.

(2). Determine the values of the standard error of estimate and the index of correlation based on the second degree parabola. (See *Text* Chapter XII.)

(3). Compare these values with the results secured for Problem 9. Discuss briefly.

11. (1). Determine the equation to a straight line [1] fitted to the data below. Compute the coefficient of correlation and the standard error of estimate.

(2). Fit a second degree parabola [1] to the same data. Determine the values of the index of correlation and the standard error of estimate.

(3). Determine the value of the correlation ratio (*Text* p. 441). Compare the values of the coefficient of correlation, the index of correlation and the correlation ratio. Compare the two standard errors of estimate.

(4). Compute the value Zeta, $(\eta^2 - r^2)$, as a test of linearity. By reference to the formula for the probable error of Zeta (*Text* page 557) determine whether the relationship between cost per bushel and yield per acre is definitely non-linear. If the regression were truly linear what are the probabilities of securing a value for Zeta as great as or greater than the one obtained?

[1] The line should be fitted to the mid-values of the separate compartments of the correlation table, weighted according to the number of cases in each compartment.

Correlation Table showing the Relation between Yield of Wheat per acre and Production Cost per bushel on 216 Farms in Oklahoma, Kansas, Nebraska and Missouri in 1920.[1]

Number of Farms with Specified Yields in Bushels per Acre

Cost per Bushel — Classes	0 to 4.99	5 to 9.99	10 to 14.99	15 to 19.99	20 to 24.99	25 to 29.99	30 to 34.99	Totals
$8 to $8.99	1	1						2
7 to 7.99								0
6 to 6.99	1	2						3
5 to 5.99		1						1
4 to 4.99	2	2	2					6
3 to 3.99		12	6					18
2 to 2.99		4	42	26				72
1 to 1.99		1	22	49	30	9	2	113
0 to .99							1	1
Totals	4	23	72	75	30	9	3	216

12. It is desired to determine the degree of relationship between the yield per acre and the cost per bushel of raising wheat on the North Dakota farms recorded in Problem H–9, as measured by the correlation ratio (*Text* pp. 441–450).

(1). Construct a correlation table and tabulate the items therein. (Cost per bushel should be considered as the dependent variable.)

(2). Calculate the values of σ_y, σ_{my} and η_{yx}.

(3). Compute Zeta, $(\eta^2 - r^2)$, the test of linearity. Does it indicate that the relationship is definitely non-linear?

13. It is desired to determine the relation between the price per car (dependent variable) and the available supply of watermelons in New York, as measured by the number of cars on track (independent variable).

For discussion of the methods employed in this problem see *Text* Chap. XIII.

[1] These data are from Department Circular 307 of the U. S. Department of Agriculture, *A Method of Testing Farm Management and Cost of Production Data for Validity of Conclusions,* by H. R. Tolley and S. W. Mendum.

(1). Plot on coördinate paper the data below. Determine the equation to a straight line fitted to the data. Compute the standard error of estimate (S_y) and the coefficient of correlation.

On the graph plot the line thus determined and indicate the limits of the zones of estimate.

(2). Fit to the data an exponential curve of the type $Y = ab^X$. (Note: Reduce the equation to logarithmic form before fitting.) Determine the value of the standard error of estimate, first in logarithmic form ($S_{\log y}$) and then in ratio form (S_r). Determine the value of the index of correlation.

On a second chart plot the individual points, draw the graph of the fitted curve, and indicate the limits of the zones of estimate.

(3). Fit to the data a hyperbolic curve of the type $\frac{1}{Y} = a + bX$. Determine the values of the standard error of estimate ($S_{\frac{1}{y}}$) and the index of correlation.

On a third chart plot the individual points, draw the graph of the fitted curve, and indicate the limits of zones of estimate.

(4). In a brief summary compare the results secured from the fitting of the three different curves, with particular attention to the relation between the zones of estimate. Draw up a summary table of your results.

Curves other than those mentioned above may be employed for this problem, e.g., of the type $y = ax^b$ or H. L. Moore's demand curve, $y = Ax^{\alpha}e^{\beta x}$.

Daily Supply and Price of Watermelons in the New York Market, 1923 [1]

Date	Supply Number of cars of watermelons on track at 8 A.M.	Price Per car of 25 lb. average melons
June 1	14	$700
4	33	605
5	24	625
6	26	685
7	24	700
11	41	625
12	66	480
14	92	375
15	90	375
18	59	350

Continued on next page

[1] The data are from *Measuring the Melon Market* by W. P. Heddon and Nathan Cherniak, a bulletin issued by the Bureau of Agricultural Economics and the Port of New York Authority (Washington, D.C., 1924).

Daily Supply and Price of Watermelons in the New York Market, 1923 — Continued

Date	Supply: Number of cars of watermelons on track at 8 A.M.	Price: Per car of 25 lb. average melons
June 20	22	$525
21	21	525
25	23	740
26	35	650
27	39	600
28	38	600
29	26	515
July 2	63	495
3	90	420
5	115	350
6	86	455
9	89	350
10	95	325
11	114	325
12	121	300
13	87	395
16	62	440
17	90	365
18	95	405
19	73	380
20	67	325
23	123	370
24	124	360
25	88	340
27	52	365
30	116	320
Aug. 2	109	280
3	100	310
6	162	210
7	143	260
8	120	275
9	86	295
13	105	310
14	92	425
15	87	350

Additional material for problems in simple correlation may be found in Section I, *problems* I–1 *and* I–2. *The relation between any two of the variables listed in these problems may be studied. It may be desirable to use this material for short laboratory exercises.*

I. Multiple and Partial Correlation

1. The table given below contains a record of (1) expenditures for local government, (2) the assessed value of real estate, (3) population and (4) area, for a group of thirty-five counties of New York State. It is desired to ascertain the relation between the last three items, severally and collectively, and the cost of local government.

(1). Determine the equation of regression, the standard error of estimate and the coefficient of correlation for each of the combinations listed below. Use governmental expenditures as the dependent variable in each case.

(*a*) Governmental expenditures and assessed value of real estate.
(*b*) Governmental expenditures and population.
(*c*) Governmental expenditures and area.

(2). Summarize these results in a table. In which case is the relationship greatest? Interpret and compare the standard errors of estimate.

(3). Calculate the equation to the line of average relationship, the standard error of estimate, and the coefficient of multiple correlation in each of the following cases:

(*a*) Governmental expenditures (dependent variable), assessed value of real estate and population (independent variables).
(*b*) Governmental expenditures (dependent variable), assessed value of real estate and area (independent variables).
(*c*) Governmental expenditures (dependent variable) population and area (independent variables).

(4). Summarize these results in tabular form. Can better estimates be made when two independent factors are considered, instead of one? How much better?

(5). Consider governmental expenditures as affected by all three factors. Determine the estimating equation, the standard error of estimate and the coefficient of multiple correlation. Does this give a better means of estimation? From the equation estimate the cost of government in each one of these counties and compare with the actual figures. What percentage of the counties falls within plus and minus 1S from the line of average relationship? plus and minus 2S? plus and minus 3S?

Factors Affecting the Cost of Local Government in Thirty-five Selected New York Counties — 1920

County	Governmental expenditures within the county (thousands of dollars) X_1	Assessed value of real estate (millions of dollars) X_2	Population (thousands) X_3	Area (thousands of acres) X_4
Alleghany	518	32	37	638
Cattaraugus	1001	47	70	802
Cayuga	943	48	65	420
Chemung	927	54	66	246
Chenango	551	20	35	549
Clinton	441	14	44	595
Columbia	588	28	39	441
Cortland	363	22	30	310
Delaware	579	27	43	877
Fulton	634	33	45	314
Genesee	566	39	38	310
Greene	355	15	26	371
Herkimer	844	48	65	879
Lewis	311	14	24	795
Livingston	488	33	37	384
Montgomery	839	37	58	227
Ontario	716	52	53	387
Orleans	339	30	29	238
Oswego	864	43	71	585
Otsego	593	36	46	609
Putnam	196	15	11	137
Rockland	699	36	46	73
Saratoga	1075	38	60	496
Schoharie	240	12	21	371
Schuyler	169	8	13	199
Seneca	370	20	25	200
Steuben	959	48	81	838
Sullivan	431	8	33	604
Tiogo	320	15	24	315
Tompkins	525	28	35	293
Ulster	897	40	75	668
Warren	567	18	32	495
Washington	593	23	45	500
Wyoming	407	26	30	370
Yates	188	13	17	206

Sources: *Report of the New York State Tax Commission. Report of the Comptroller of New York State.*

2. The following table presents data relating to land values and crops in certain Iowa counties: [1]

Data from Twenty-five Iowa Counties

County	Average value per acre of farm land Jan. 1, 1920 X_1	Average corn yield per acre, 1910–1919, in bu. X_2	Percent of farm land in small grain X_3	Percent of farm land in corn X_4	Average No. of improved acres per farm X_5
Allamakee..	$ 87	40	11	14	103
Bremer.....	133	36	13	30	102
Butler.....	174	34	19	30	137
Calhoun....	285	41	33	39	160
Carroll.....	263	39	25	33	157
Cherokee...	274	42	23	34	166
Dallas.....	235	40	22	37	130
Davis.....	104	31	9	20	119
Fayette....	141	36	13	27	106
Fremont ...	208	34	17	40	137
Howard....	115	30	18	19	136
Ida.....	271	40	23	31	185
Jefferson ...	163	37	14	25	98
Johnson....	193	41	13	28	122
Kossuth....	203	38	24	31	173
Lyon.....	279	38	31	35	182
Madison ...	179	34	16	26	124
Marshall ...	244	45	19	34	138
Monona....	165	34	20	30	148
Pocahontas.	257	40	30	38	164
Polk.....	252	41	22	35	96
Story.....	280	42	21	41	132
Wapello....	167	35	16	23	96
Warren.....	168	33	18	24	118
Winneshiek.	115	36	18	21	113

(*These data may be used in various combinations for simple, multiple and partial correlation problems. The problem outlined below deals only with the first four variables, but if the time devoted to this subject permits, the other variable may be introduced to advantage.*)

(1). (*a*) By the method of least squares derive the equation which describes the average relationship between variables X_1 (dependent) and X_2, X_3, X_4. (The necessary values should be expressed in terms of

[1] This table is taken from a valuable pamphlet on *Correlation and Machine Calculation*, by H. S. Wallace and G. W. Snedecor, published by Iowa State College of Agriculture and Mechanic Arts.

deviations from the mean in preparing the normal equations, in order to facilitate the computations. It is suggested that the Doolittle method be employed in solving the simultaneous equations. (*Text* pp. 578–581.)

(*b*) Compute the standard error of estimate.

(*c*) Compute the coefficient of multiple correlation.

(2). It is desired to determine the coefficient of net or partial correlation between the average value per acre and each of the three independent variables (i.e., the values of $r_{12.34}$, $r_{13.24}$ and $r_{14.23}$ are required).

(*a*) Compute the zero order coefficients of correlation (i.e., r_{12}, r_{13}, r_{14}, r_{23}, r_{24}, r_{34}).

(*b*) Compute the required first order coefficients from the zero order coefficients. This should be done in the systematic fashion illustrated in the *Text*, p. 510. Miner's *Tables* or, if these are not available, Table VIII of Pearson's *Tables*, may be utilized.

(N.B. The framework for final calculation of the required second order coefficients should be laid out before the first order coefficients are computed. This is advisable in order that only those coefficients which are required for the subsequent calculations be computed.)

(*c*) Compute the three required second order coefficients of correlation. (Each of these should be derived by two different processes for the purpose of checking the calculations.)

(*d*) In a brief summary interpret all your results.

J. Simple Probabilities. Further Measures Descriptive of the Frequency Distribution. Probable Errors of Statistical Measures.

1. What is the probability of drawing a spade from a pack of fifty-two cards? Of drawing two spades in succession (assuming that the first card is replaced before the second is drawn)? Of drawing an ace? Of drawing three aces in succession (assuming that the card drawn is replaced each time before the next is drawn)?

2. What is the probability of drawing an ace *or* a two-spot in one draw from a pack of fifty-two cards?

3. What is the probability of throwing exactly seven in the toss of two dice? Of throwing seven *or* eleven in a single toss of two dice? Of throwing seven *or more* in a single toss of two dice?

4. In an experiment described by Joseph Cohen [1] twenty coins were tossed 800 times.

The results are presented in the following table:

[1] *Chance: A Comparison of Facts with the Theory of Probabilities.* London, C. & E. Layton. 46.

Distribution of Results in Coin Tossing Experiment
(20 coins tossed 800 times)

Number of heads	Frequency of occurrence
Three	1
Four	2
Five	8
Six	23
Seven	66
Eight	91
Nine	130
Ten	158
Eleven	125
Twelve	87
Thirteen	66
Fourteen	31
Fifteen	10
Sixteen	2
	800

(1). Substituting the proper values in the formula $N(p + q)^n$ compute the theoretical frequencies for the above problem. (Note: In this case $N = 800$; $n = 20$; $p = \frac{1}{2}$; $q = \frac{1}{2}$.)

Construct on a single chart frequency polygons showing the actual and the theoretical frequencies.

(2). From the formula $M = np$ compute the theoretical mean value of the number of heads appearing. Compare this with the actual mean value, as computed from the frequency table.

(3). From the formula $\sigma = \sqrt{npq}$ compute the theoretical standard deviation. Compare this with the standard deviation computed from the distribution of actual results.

5. The following table summarizes the results secured by Joseph Cohen from a similar experiment:

Distribution of Results in Coin Tossing Experiment
(8 coins tossed 500 times)

Number of heads	Frequency of occurrence
None	1
One	15
Two	52
Three	112
Four	143
Five	117
Six	45
Seven	13
Eight	2
	500

For this distribution compute the measures and make the comparisons suggested for the preceding table.

6. It was found that the distribution of residence telephone subscribers in Buffalo, N. Y., classified according to message use, followed the normal law of error (*Text* Chap. XV). Using the figures given in the *Text*, and referring to the table of areas under the normal curve (Table F, section VIII of this *Manual*) answer the following questions:

(1). What is the probability that, in the population to which this sample of telephone subscribers belongs, there will be found a residence subscriber having more than 1200 messages per year? More than 1150?

(2). What is the probability that a subscriber picked at random will have registered more than 300 and less than 400 calls in a year? More than 800 and less than 850? More than 600 and less than 610?

(3). If another sample of 400 subscribers is taken from the same population, how many may be expected to have registered between 200 and 250 calls? Between 660 and 680 calls? More than 700 calls?

7. *Distribution of London-New York Exchange Rates as Recorded by Months during the Period* 1882–1913

Class-Interval	Frequency (Number of months given rate prevailed)
$4.8275–$4.8324	1
4.8325– 4.8374	6
4.8375– 4.8424	11
4.8425– 4.8474	21
4.8475– 4.8524	23
4.8525– 4.8574	24
4.8575– 4.8624	25
4.8625– 4.8674	40
4.8675– 4.8724	45
4.8725– 4.8774	49
4.8775– 4.8824	35
4.8825– 4.8874	45
4.8875– 4.8924	33
4.8925– 4.8974	16
4.8975– 4.9024	8
4.9025– 4.9074	1
4.9075– 4.9124	1
	384

(1). Construct a column diagram based upon this table.

(2). Calculate the first four moments of the distribution about an arbitrary origin. (*Text* p. 528 *et seq.*)

(3). Calculate the first four moments about the mean, applying Sheppard's corrections.

(4). Compute β_1, β_2 and κ_2.

(5). Fit a normal curve to the distribution, using Table F, section VIII of this *Manual*, giving ordinates of the normal curve in terms of abscissa.

(6). Tabulate the theoretical frequencies against the actual frequencies. Plot the actual data and the fitted normal curve as frequency polygons.

(7). Using the table of areas in Section VIII, compute the theoretical frequencies.

(8). Pick out two classes for which actual frequencies and frequencies as computed from the table of areas differ most widely. Determine the standard error of sampling for each of these classes. What is the probability, in each of these cases, that the difference is due to chance fluctuations of sampling, assuming that the underlying distribution is truly described by the normal curve?

(9). Apply the Chi-square test of the goodness of the fit of the normal curve to this distribution. In how many times out of 1000 trials would one obtain a fit as bad as or worse than this one, if the distribution were normal?

(10). Compute a measure of skewness based on β_1 and β_2.

(11). Compute a measure of kurtosis.

(12). Compute d (the difference between mean and mode). Determine the value of the mode.

8. A detailed comparison of the two frequency distributions of Illinois coal miners given in Problem E–8 of this *Manual* is desired. For each of these distributions compute the first four moments about the mean. (*Text* p. 528 *et seq.*) From these values calculate:

(1). the arithmetic mean
(2). the standard deviation
(3). the criteria β_1, β_2 and κ_2
(4). a measure of skewness (*Text* p. 546)
(5). a measure of kurtosis (*Text* p. 545)
(6). the value of the mode (*Text* p. 546)

In a brief summary discuss the significance of these measures for the two distributions.

(Similar problems may be given, using the data of any of the frequency distributions presented in Section E. For advanced work students may be required to compute the probable errors of the criteria of curve type to de-

termine the type of ideal frequency curve to be fitted in each case. The formulas for the probable errors of the criteria of curve type will be found in Pearson's ***Tables,*** *and methods of fitting are illustrated in W. P. Elderton's* ***Frequency Curves and Correlation.***)

9. You are given the following information concerning a frequency distribution, which follows, approximately, the normal law:

$$M = 39.71¢$$
$$Md = 40.05¢$$
$$Q_1 = 31.72¢$$
$$Q_3 = 47.53¢$$
$$\sigma = 11.93¢$$
$$N = 678$$

Determine the standard error of each of the above measures. Comment briefly on the significance of the standard error of the mean and the standard error of the standard deviation.

Of the two given measures of central tendency, which is the more reliable?

10. In studying the relation between discount rates of Federal Reserve banks and discount rates of commercial banks, a coefficient of correlation of + .82 was derived. The number of observations was 504. (*Text* Chap. X.)

What is the probable error of this measure? Interpret this figure.

11. The coefficient of multiple correlation between corn yield per acre in Kansas and temperature during the three months, June, July and August, was found to have a value of .778. The number of observations was 33. (*Text* Chap. XIV.)

Determine the probable error of this measure.

12. The coefficient of regression derived in measuring the relation between automobile registration and income in 43 states has a value of 1.91 (*Text* p. 367). What is the probable error of this measure? Interpret this probable error.

13. In studying the relation between wheat yield and fertilizer applied (*Text* p. 443) the following measures were secured:

$$r = .793$$
$$\eta = .964$$

These were based upon 193 experiments.

Determine:

(*a*) the probable error of r
(*b*) the probable error of η
(*c*) the value of ζ, the test of linearity
(*d*) the probable error of ζ

If the regression were truly linear, what are the probabilities of securing a value of ζ as great as or greater than that obtained?

14. Problem E–8 in this *Manual* called for the comparison of two frequency distributions, one relating to the earnings of coal miners in southern Illinois, the other to the earnings of coal miners in central Illinois. Is the difference between the arithmetic means of these two distributions great enough to indicate that miners in these two sections do not belong to a single homogeneous group?

(Note: Apply the formula for the standard error of the difference between two means.)

(*Exercises similar to those above, calling for the calculation of the probable errors of the chief statistical measures, may be derived from all the problems relating to frequency distributions and the measurement of correlation given in the preceding pages of this* **Manual.** *Practice in the calculation and careful interpretation of these results will be useful to the student, but care should be taken that the assumptions upon which these formulas rest are not overlooked.* [***Text,*** *Chap. XVI*].)

K. Miscellaneous Problems

1. The Rutherford Manufacturing Company, whose record is appended below, is engaged in making a type of heavy fire-proof roofing principally used in industrial construction. You are called upon to make a complete analysis of these figures and to determine the influence of the business cycle on this concern.

Monthly Record of Orders for Rutherford Fire-proof Roofing, 1911–1924

Date	1911	1912	1913	1914	1915	1916	1917
January....	$14,209	$30,598	$59,917	$20,215	$24,985	$112,553	$175,668
February...	13,927	29,564	57,275	27,911	36,028	106,424	146,130
March.....	33,998	56,593	61,737	58,248	93,307	180,823	317,360
April.....	46,466	30,386	55,050	61,087	85,104	107,988	208,007
May.....	48,657	58,062	77,890	65,921	113,222	188,661	259,722
June.....	24,785	58,593	73,863	68,686	164,792	130,809	234,904
July.....	65,245	81,185	79,072	83,073	120,442	127,768	247,958
August.....	64,367	97,277	77,497	155,723	160,683	226,047	255,972
September..	52,064	48,853	65,990	53,100	168,890	211,202	236,326
October....	46,636	70,758	37,750	49,735	153,964	206,536	275,212
November..	12,585	63,809	68,705	55,657	143,121	233,956	346,214
December..	28,346	54,372	43,831	49,344	177,563	423,908	362,776
Total	$451,285	$680,050	$758,577	$748,700	$1,442,101	$2,256,675	$3,066,249

Continued on next page.

Date	1918	1919	1920	1921	1922	1923	1924
January....	$154,628	$66,489	$216,529	$78,159	$103,211	$316,558	$171,389
February...	140,684	65,316	601,802	79,661	97,168	247,003	153,692
March.....	204,778	78,865	514,405	103,892	108,052	398,124	280,891
April.....	296,588	211,139	464,866	154,610	175,840	455,508	278,071
May.....	244,936	202,272	593,779	115,873	212,923	440,535	280,413
June.....	305,125	234,244	705,227	143,092	365,814	321,591	263,978
July.....	308,592	328,296	412,122	132,875	251,848	350,275	271,013
August.....	317,005	441,830	636,195	118,241	393,894	379,843	280,059
September..	312,724	791,998	518,967	150,416	283,414	343,960	230,257
October....	277,077	889,259	476,059	171,781	320,013	297,865	290,005
November..	170,968	795,384	310,812	134,283	260,420	250,869	360,988
December..	177,421	892,651	223,494	146,108	188,993	265,973	450,441
Total	$2,910,526	$4,997,743	$5,674,257	$1,528,991	$2,761,590	$4,068,104	$3,311,197

As the figures are given in dollars it would be advisable to deflate them, i.e., to remove the fluctuations due to changes in prices. (*Text* p. 307.) The records show that this company's prices have changed in conformity with the following index numbers (1913 = 100):

Jan. 1911	95	June 1914	100	Jan. 1918	125	June 1921	120
June 1911	95	Jan. 1915	105	June 1918	135	Jan. 1922	130
Jan. 1912	98	June 1915	110	Jan. 1919	150	June 1922	135
June 1912	98	Jan. 1916	115	June 1919	180	Jan. 1923	140
Jan. 1913	100	June 1916	120	Jan. 1920	190	June 1923	140
June 1913	100	Jan. 1917	125	June 1920	170	Jan. 1924	150
Jan. 1914	100	June 1917	135	Jan. 1921	140	June 1924	150

What do the deflated figures represent?

Determine the equation to an appropriate line of trend for Rutherford orders. On the basis of this trend make an estimate of the orders for 1925.

Calculate an index of seasonal variation. On this basis what orders would you estimate for each month of 1925?

Eliminate the effects of trend and of seasonal variation from the deflated figures. How do these figures, when plotted, compare with the composite index of general business conditions given on page 356 of the *Text?*

Write a brief report summarizing the results of your analysis.

2. The Blank Sales Company is engaged in marketing an article of domestic use. It appeals to consumers through the medium of three or four magazines with a national circulation. Its product is marketed by means of a house to house canvass except in New York and Chicago, where it has been found more economical to sell through retail stores.

Determine the value of the *Literary Digest* list of home telephone

subscribers as an index of potential demand upon which to base sales quotas in those states where house to house canvasses are made (i.e., except in New York and Illinois). The following table shows the number of home telephone subscribers on the *Digest* list and the gross sales of the Blank Company in 1924:

State	Number of subscribers to the *Literary Digest* having telephones in their homes. (in thousands)	Blank Sales Company gross sales in 1924 (in thousands)
Alabama	51.2	$ 155
Arizona	13.3	41
Arkansas	54.3	180
California	445.3	1,387
Colorado	86.0	254
Connecticut	144.9	478
Delaware	13.2	47
Dist. of Col.	50.1	140
Florida	43.2	143
Georgia	83.5	266
Idaho	26.8	84
Illinois	855.4	2,520
Indiana	322.2	997
Iowa	317.1	972
Kansas	179.9	571
Kentucky	87.7	222
Louisiana	58.7	231
Maine	77.0	270
Maryland	89.8	229
Massachusetts	461.1	1,428
Michigan	310.9	970
Minnesota	238.8	759
Mississippi	35.7	120
Missouri	285.1	825
Montana	77.9	256
Nebraska	158.3	449
Nevada	4.8	10
New Hampshire	48.4	109
New Jersey	246.5	708
New Mexico	9.4	12
New York	1,037.7	3,923
North Carolina	60.6	230
North Dakota	40.0	119
Ohio	572.4	1,726
Oklahoma	128.6	390

Continued on next page.

State	Number of subscribers to the *Literary Digest* having telephones in their homes. (in thousands)	Blank Sales Company gross sales in 1924 (in thousands)
Oregon	89.6	$ 273
Pennsylvania	575.6	1,800
Rhode Island	58.3	167
South Carolina	25.3	83
South Dakota	44.8	128
Tennessee	87.7	261
Texas	248.2	975
Utah	30.8	101
Vermont	33.8	105
Virginia	76.5	219
Washington	136.6	400
West Virginia	64.0	160
Wisconsin	268.4	879
Wyoming	12.2	13

3. The table on page 111 shows the monthly sales of the Ashford Tire Company.

There is required a complete analysis of these figures in order to determine the character of the forces causing fluctuations in these sales and to derive, if possible, a method of forecasting the major fluctuations. The following operations should be performed:

(1). Deflate the sales, using an appropriate deflating index.
(2). Fit a line of trend to the deflated data.
(3). Compute index numbers of seasonal variation. (By inspection, or, if necessary, by actual computation, test to see if the seasonal movement has varied.)
(4). Compute the deviations of the actual monthly sales from normal, correcting for seasonal variation. Construct a chart showing these corrected deviations.
(5). Compare this graph with similar graphs of standard index numbers of business conditions, or of basic business series believed to be related to the sales of this company (e.g., The A. T. & T. composite index of business activity, as given in the *Text* p. 358; the average price of industrial stocks, as given in *Text* p. 421; the general business curve of the Harvard Economic Service; the index of production in basic industries of the Federal Reserve Board; the price of raw rubber; automobile sales).
(6). Select the index series which seem to be most closely related in its cyclical movements to this company's sales, preferably

one which appears to move in advance of the sales. By means of the coefficient of correlation determine the precise time relation between the cyclical movements of sales and the cyclical movements of the index in question.

Gross Sales of the Ashford Tire Company, 1908–1924

(Unit: one thousand dollars)

	1908	1909	1910	1911	1912	1913	1914	1915	1916
January	8	24	62	53	118	189	187	163	460
February	11	24	71	83	150	202	182	191	380
March	19	38	86	116	213	230	273	250	490
April	24	41	111	134	281	319	336	404	653
May	25	41	123	139	297	398	293	349	529
June	26	39	119	136	280	394	294	378	624
July	25	46	93	141	247	334	298	387	619
August	23	48	78	139	253	320	329	431	673
September	17	47	61	150	254	313	291	422	683
October	20	41	58	142	239	319	253	407	658
November	15	39	49	90	124	165	132	307	530
December	19	50	46	96	159	192	131	335	581

	1917	1918	1919	1920	1921	1922	1923	1924
January	690	680	800	1,820	620	580	820	909
February	601	720	870	1,707	640	642	960	1,028
March	830	780	980	2,160	690	769	1,483	1,202
April	1,007	710	893	1,703	640	721	1,520	900
May	1,030	850	1,021	1,907	750	803	1,897	843
June	1,159	1,103	1,153	2,003	800	921	2,033	850
July	1,218	1,050	1,321	1,805	904	880	1,864	880
August	1,257	1,321	1,242	1,906	867	976	1,732	600
September	1,164	1,223	1,461	1,801	803	937	1,200	592
October	1,135	1,407	1,390	1,700	902	981	925	480
November	986	1,350	1,600	1,608	997	940	800	505
December	890	1,281	1,400	700	861	886	896	470

4. A detailed analysis of the following sales records is to be made. This should include:

(1). Determination of the general trends of sales and selling expenses.

(2). Determination of the average value of orders received and of the average selling cost per order, together with the trends, over the period given, of these two values.

(3). Comparison of districts with respect to the present situation and the past trends in the value of sales, amount of goods sold, number of orders, and selling expenses. The relative

efficiency of the selling methods employed in the different districts should be determined by comparing sales with population figures or with some other index of potential demand.

(4). Determination of the degree to which the profitable lines have been developed in the different districts.

(5). Comparison of different districts with respect to the degree of development of urban and rural territory.

Write out a brief summary of your conclusions, accompanying your report with graphs bringing out all the salient facts revealed by your analysis.

On the basis of this analysis, draw up sales quotas, by districts, for 1925, with brief instructions to the district sales managers.

Wholesale Grocery Sales of the XYZ Company, by Districts, 1910–1924

District #1

	No. of orders	Value of sales (thousands)	Selling expenses (thousands)
1910	76,200	$12,800	$340
1911	80,000	13,800	350
1912	82,000	15,000	320
1913	82,500	14,500	380
1914	84,000	14,600	400
1915	82,000	16,000	420
1916	90,000	18,400	500
1917	95,000	23,800	600
1918	98,000	27,600	600
1919	102,000	32,100	620
1920	108,000	33,000	580
1921	80,000	19,000	400
1922	87,000	23,000	450
1923	96,000	24,000	440
1924	89,000	20,000	460

District #2

1910	14,800	$ 7,400	$160
1911	15,000	7,300	170
1912	14,200	7,800	200
1913	14,800	8,600	200
1914	15,300	9,400	220
1915	16,000	9,800	220
1916	16,200	11,200	250
1917	16,400	14,000	330
1918	16,400	18,000	390
1919	16,600	18,500	470

	No. of orders	Value of sales (thousands)	Selling expenses (thousands)
1920	16,600	$17,500	$360
1921	14,000	15,000	400
1922	15,000	18,000	500
1923	16,700	20,000	550
1924	16,800	19,000	550

District #3

1910	29,000	$38,000	$740
1911	30,000	35,000	702
1912	30,500	34,500	790
1913	30,300	32,100	800
1914	29,500	33,200	830
1915	29,000	32,600	815
1916	30,000	37,200	940
1917	30,400	49,000	1,020
1918	31,000	56,000	1,350
1919	31,800	68,000	1,650
1920	32,000	53,000	1,260
1921	26,000	40,000	1,200
1922	26,500	41,000	1,250
1923	26,000	35,000	1,300
1924	25,000	30,000	1,300

Classification of Merchandise Sold by Districts in 1924

(in percent)

	Class A	Class B	Class C	Class D	Total
District #1	38	5	50	7	100
District #2	30	12	43	15	100
District #3	26	15	39	20	100

The following are the classes of merchandise named above with the average percentage of gross profit secured by the dealer in question in each line:

Class A — Canned goods. Profit 20 %
Class B — Sugar, spices, baking powder, etc. Profit 5 %
Class C — Breakfast foods. Profit 25 %
Class D — Soap, washing powder, etc. Profit 10 %

The geographic limits of the sales districts are as follows:

District #1 Massachusetts
District #2 Connecticut and Rhode Island
District #3 Pennsylvania

The Bureau of Labor Statistics index number of wholesale food prices is as follows (1913 = 100):

1910–101	1915–105	1920–220
1911– 97	1916–121	1921–144
1912–104	1917–167	1922–138
1913–100	1918–188	1923–144
1914–102	1919–207	1924–144

Urban and Rural Distribution of 1924 Sales

	Percentage of sales in urban territory (in towns of population of 2,500 and over)	Percentage of sales in rural territory
District #1	75	25
District #2	65	35
District #3	80	20

5. A study of the production, consumption and price of an assigned commodity is to be made, in order that a purchasing policy may be formulated. The specific objects of the study are:

(1). To ascertain supply conditions and trends.
  (*a*) Production in U. S.; number of producing plants.
  (*b*) Imports, with statement of chief countries from which commodity is imported.
  (*c*) Probable trends of production and imports.

(2). To ascertain consumption conditions and trends.
  (*a*) Chief uses and chief consuming industries.
  (*b*) Amount consumed in U. S.
  (*c*) Amount exported, with names of chief countries to which commodity is exported.
  (*d*) Probable trends of domestic consumption and exports.

(3). To ascertain price conditions and trends.
  (*a*) Wholesale prices in the chief domestic market.
  (*b*) Seasonal variation in price, if any.
  (*c*) Relation between price trend and production trend; between price trend and import trend.
  (*d*) Relation of price trend of this commodity to general price trend.
  (*e*) Probable price movement of this commodity during the next cycle, and over an extended period.

The three sets of figures indicated above are to be obtained over a 15 year period, and plotted on small coördinate paper. The following charts are required:

(1). A production chart, with an extension to show probable future production.
(2). An import chart, with extension to show trend.
(3). A chart showing domestic consumption, with extension to show trend.
(4). An export chart, with extension to show trend.
(5). A chart showing the course of prices of the commodity in question in comparison with the course of general wholesale prices.
(6). A chart showing seasonal variations in the price of the commodity.

On the basis of this study the following questions are to be answered:

(1). Should a short or long term contract be made at prevailing prices?
(2). What is the best season of the year for buying?
(3). Will the normal supply be adequate for domestic and foreign requirements or is a condition of scarcity probable in the future?

(The chief sources of information utilized are to be named in the report.)

The commodities to be assigned in connection with this problem may be selected by the instructor from a list of basic raw materials of industrial importance.

6. Each student will be assigned a monthly time series of economic interest for intensive study. Any of those given in Section IV, or any available elsewhere, may be chosen.

(1). Plot the series, by months, from 1913 to date.
(2). Fit a line of trend.
(3). Measure the seasonal variation.
(4). Measure the cyclical movements of the series in question.
(5). By referring to the index of industrial stock prices (*Text* p. 423), and to the composite index of general business activity (*Text* p. 356), point out the characteristic fluctuations of the series during the business cycle.
(6). Briefly describe the present condition of this series, with particular reference to the probable future trend, and indicate the bearing of this condition upon the course of general business.
(7). Write a brief forecast of general business conditions for the next twelve months.

7. Select one of the business forecasting services for study, and submit a report giving detailed information on the following points:

(1). Character of service.
(2). Types of reports regularly published.
(3). Methods used in forecasting.

IV. SELECTED

Annual Production of

Year	Pig Iron (1,000,000 long tons) (1)	Coal Bituminous (1,000,000 short tons) (2)	Coal Anthracite (1,000,000 short tons) (3)	Coke (1,000,000 short tons) (4)	Petroleum (1,000,000 barrels) (5)
1890	9.20	111.30	46.47	11.51	45.8
1891	8.28	117.90	50.67	10.35	54.3
1892	9.16	126.86	52.47	12.01	50.5
1893	7.12	128.39	53.97	9.48	48.4
1894	6.66	118.82	51.92	9.20	49.3
1895	9.45	135.12	58.00	13.33	52.9
1896	8.62	137.64	54.35	11.79	61.0
1897	9.65	147.61	52.61	13.29	60.5
1898	11.77	166.59	53.38	16.05	55.4
1899	13.62	193.32	60.42	19.67	57.1
1900	13.79	212.32	57.37	20.53	63.6
1901	15.88	225.83	67.47	21.80	69.4
1902	17.82	260.22	41.37	25.40	88.8
1903	18.01	282.75	74.61	25.27	100.5
1904	16.50	278.66	73.16	23.66	117.1
1905	22.99	315.06	77.66	32.23	134.7
1906	25.31	342.87	71.28	36.40	126.5
1907	25.78	394.76	85.60	40.78	166.1
1908	15.94	332.57	83.27	26.03	178.5
1909	25.80	379.74	81.07	39.32	183.2
1910	27.30	417.11	84.49	41.71	209.6
1911	23.65	405.91	90.46	35.55	220.4
1912	29.73	450.10	84.36	43.98	222.9
1913	30.72	478.44	91.52	46.30	248.4
1914	23.05	422.70	90.82	34.56	265.8
1915	29.66	442.62	89.00	41.58	281.1
1916	39.04	502.52	87.58	54.53	300.8
1917	38.18	551.79	99.61	55.61	335.3
1918	38.51	579.39	98.83	56.48	355.9
1919	30.59	465.86	88.09	44.18	378.4
1920	36.42	568.67	89.60	51.35	442.9
1921	16.55	415.92	90.47	25.29	469.6
1922	26.88	422.27	54.68	37.12	557.6
1923	40.06	564.16	95.45	55.49	732.4
1924	31.10	468.96	90.22	43.45	707.3
1925	.....	.....	.....	.....	.....
1926	.....	.....	.....	.....	.....
1927	.....	.....	.....	.....	.....

TIME SERIES

Important Commodities

Portland Cement (1,000,000 barrels) (6)	Copper (1,000,000 pounds) (7)	Wheat (1,000,000 bushels) (8)	Corn (1,000,000 bushels) (9)	Cotton (1,000 bales) (10)	Wool (1,000 pounds) (11)	Year
.34	260	378	1,460	8,674	276,000	1890
.45	284	585	2,055	9,018	285,000	1891
.55	345	528	1,713	6,664	294,000	1892
.59	329	428	1,707	7,493	348,538	1893
.80	354	516	1,339	9,476	325,211	1894
.99	381	569	2,310	7,161	294,297	1895
1.54	460	544	2,503	8,533	272,475	1896
2.68	494	610	2,144	10,898	259,153	1897
3.69	526	772	2,261	11,189	266,721	1898
5 65	569	636	2,454	9,345	272,191	1899
8.48	606	603	2,505	10,123	288,637	1900
12.71	602	789	1,613	9,510	302,502	1901
17.23	660	725	2,619	10,631	316,341	1902
22.34	698	664	2,346	9,851	287,450	1903
26.51	812	597	2,528	13,438	291,783	1904
35.25	889	727	2,748	10,575	295,488	1905
46.46	918	757	2,897	13,274	298,915	1906
48.79	869	638	2,512	11,107	298,295	1907
50.07	943	645	2,544	13,242	311,138	1908
64.99	1,093	700	2,572	10,005	328,111	1909
76.55	1,080	635	2,886	11,609	321,363	1910
78.53	1,097	621	2,531	15,693	318,548	1911
82.44	1,243	730	3,124	13,703	304,043	1912
91.44	1,224	763	2,446	14,156	296,175	1913
87.48	1,150	891	2,672	16,135	290,192	1914
85.80	1,388	1,026	2,994	11,192	285,726	1915
91.08	1,928	636	2,566	11,450	288,490	1916
92.64	1,886	637	3,065	11,302	281,892	1917
70.80	1,909	921	2,502	12,041	298,870	1918
80.40	1,286	968	2,811	11,421	298,258	1919
99.72	1,209	833	3,208	13,440	277,905	1920
98.28	575	815	3,068	7,954	271,562	1921
113.88	1,131	868	2,906	9,762	264,560	1922
137.40	1,667	797	3,054	10,140	266,110	1923
148.80[p]	1,796	873	2,437	13,153	.....	1924
.....	.....	.....	.....	.....	.....	1925
.....	.....	.....	.....	.....	.....	1926
.....	.....	.....	.....	.....	.....	1927

[p] Preliminary

Year	Total Imports of Merchandise ($1,000,000) (12)	Exports of Merchandise ($1,000,000) (13)	Imports of Crude Rubber (1,000,000 Pounds) (14)	Bank Clearings New York City ($1,000,000,000) (15)
1890	789	858	33.8	37.5
1891	845	884	33.7	33.7
1892	827	1,030	40.0	36.7
1893	866	848	41.5	31.3
1894	655	892	33.8	24.4
1895	732	808	39.7	29.8
1896	780	883	36.8	28.9
1897	765	1,051	35.6	33.4
1898	616	1,231	46.1	42.0
1899	697	1,227	51.1	60.8
1900	850	1,394	49.4	52.6
1901	823	1,488	55.3	79.4
1902	903	1,382	50.4	76.3
1903	1,026	1,420	55.0	66.0
1904	991	1,461	59.0	68.6
1905	1,118	1,519	67.2	93.8
1906	1,227	1,744	57.8	104.7
1907	1,434	1,881	77.0	87.2
1908	1,194	1,861	62.2	79.3
1909	1,312	1,663	88.4	103.6
1910	1,557	1,745	101.0	97.3
1911	1,527	2,049	72.0	92.4
1912	1,653	2,204	110.2	100.8
1913	1,813	2,466	113.4	94.7
1914	1,894	2,365	132.0	83.0
1915	1,674	2,769	172.1	110.6
1916	2,198	4,333	267.8	159.6
1917	2,659	6,290	333.4	177.4
1918	2,946	5,920	389.6	178.6
1919	3,096	7,232	402.5	235.8
1920	5,238	8,109	632.4	243.1
1921	3,654	6,516	357.0	194.4
1922	2,608	3,771	568.4	217.9
1923	3,781	3,957	797.7	214.0
1924	3,554	4,312	617.1	249.6
1925	.....	.....	.....	.....
1926	.....	.....	.....	.....
1927	.....	.....	.....	.....

SOURCES: (1) American Iron and Steel Institute. *Iron Age.* (2), (3), (4), (5), (6), (7) *Mineral Resources of the United States. Survey of Current Business.* (8), (9), (10), (11) *Agriculture Yearbook.* Standard Daily Trade Service. (12), (13), (14) *Foreign Commerce and Navigation of the United States. Monthly Summary of Foreign Commerce. Statistical Abstract of the United States.* Fiscal Years ending June 30th.

Index Numbers of Wholesale Prices		Number of Commercial Failures Dun's	Interest Rates Average Monthly Rates on 60–90 Day Commercial Paper	Stock Prices Dow, Jones Average of 20 Industrial Stocks. Year's Range Dollars per Share		Year
Bureau of Labor Statistics 1913 = 100	Bradstreet's			High	Low	
(16)	(17)	(18)	(19)	(20)	(21)	
81	.....	10,907	5.75	.....	.....	1890
80	.....	12,273	5.39	.....	.....	1891
75	$7.78	10,344	4.11	.....	.....	1892
77	7.54	15,242	6.66	.....	.....	1893
69	6.69	13,885	3.04	.....	.....	1894
70	6.43	13,197	3.66	.....	.....	1895
67	5.91	15,088	5.82	.....	.....	1896
67	6.12	13,351	3.53	$55.82	$38.49	1897
70	6.57	12,186	3.83	60.97	42.00	1898
75	7.21	9,337	4.12	77.61	58.27	1899
81	7.88	10,774	4.39	71.04	52.96	1900
79	7.58	11,002	4.28	78.26	61.52	1901
84	7.88	11,615	4.92	68.44	59.57	1902
86	7.85	12,069	5.47	67.70	42.15	1903
86	7.84	12,199	4.21	73.23	46.41	1904
86	8.10	11,520	4.40	96.56	68.76	1905
89	8.42	10,682	5.68	103.00	85.18	1906
94	8.91	11,725	6.36	96.37	53.00	1907
90	8.01	15,690	4.38	88.38	58.62	1908
97	9.35	12,924	3.98	100.53	79.91	1909
101	8.99	12,652	5.00	98.34	73.62	1910
93	8.71	13,441	4.03	87.06	72.94	1911
99	9.19	15,452	4.74	94.15	80.15	1912
100	9.21	16,037	5.60	88.57	72.11	1913
98	8.90	18,280	4.78	83.43	53.17	1914
101	9.85	22,156	3.45	99.21	54.22	1915
127	11.82	16,993	3.43	110.15	84.96	1916
177	15.64	13,855	4.74	99.18	65.95	1917
194	18.71	9,982	5.87	89.07	73.38	1918
206	18.66	6,451	5.42	119.62	79.15	1919
226	18.81	8,881	7.37	109.88	66.75	1920
147	11.37	19,652	6.53	81.50	63.90	1921
149	12.12	23,676	4.43	103.43	78.59	1922
154	13.40	18,718	4.98	105.38	85.76	1923
150	12.86	20,615	3.91	120.51	88.33	1924
.....	.....	.....	.....	.....	.....	1925
.....	.....	.....	.....	.....	.....	1926
.....	.....	.....	.....	.....	.....	1927

(15) *Financial Review. Commercial and Financial Chronicle.* (16) *Wholesale Prices,* Bulletin No. 367 of the Bureau of Labor Statistics. (17) *Bradstreet's.* (18) *Dun's Review.* (19) Federal Reserve Board. Standard Daily Trade Service. (20), (21) *Wall Street Journal.* Prior to Dec. 12, 1914 the list consisted of 12 industrials.

Year and Month	Unfilled Orders U. S. Steel Corp. (10,000 Long Tons) (1)	Gasoline Production in U. S. (1,000,000 Gallons) (2)	Pig Iron Production. Coke and Anthracite Pig Iron in the U. S. (1,000 Long Tons) (3)	Total Coke Production U. S. (1,000 Net Tons) (4)	Freight Car Loadings. Weekly Average (1,000 Cars) (5)
1913 Jan.	783	.....	2,795	4,205	.....
Feb.	766	.....	2,586	3,910	.....
Mar.	747	.....	2,764	4,042	.....
Apr.	698	.....	2,753	3,926	.....
May	632	.....	2,822	4,056	.....
June	581	.....	2,629	3,875	.....
July	540	.....	2,561	3,927	.....
Aug.	522	.....	2,546	3,954	.....
Sept.	500	.....	2,506	3,869	.....
Oct.	451	.....	2,546	3,963	.....
Nov.	440	.....	2,233	3,492	.....
Dec.	428	.....	1,984	3,081	.....
Total or Average	A 591	.....	A 2,560	T 46,300	.....
1914 Jan.	461	.....	1,885	3,003	.....
Feb.	503	.....	1,889	2,990	.....
Mar.	465	.....	2,348	3,685	.....
Apr.	428	.....	2,270	3,330	.....
May	400	.....	2,093	2,969	.....
June	403	.....	1,918	2,810	.....
July	416	.....	1,958	2,931	.....
Aug.	421	.....	1,995	2,898	.....
Sept.	379	.....	1,883	2,769	.....
Oct.	346	.....	1,778	2,607	.....
Nov.	332	.....	1,518	2,193	.....
Dec.	384	.....	1,516	2,371	.....
Total or Average	A 412	.....	A 1,921	T 34,556	.....

See note on "Sources" pages 134, 135.

Number of Business Failures, Dun's (6)	Bank Clearings New York City ($1,000,000,000) (7)	Portland Cement Production, U. S. (1,000,000 Barrels) (8)	Bureau of Labor Statistics Index Number of Wholesale Prices (1913 = 100) (9)	Domestic Cotton Consumption (1,000 Running Bales) (10)	Average Weekly Earnings in Representative New York State Factories (Dollars per Man) (11)
1,814	9.34	6.03	100	509.7	.....
1,454	7.79	5.56	100	448.1	.....
1,190	7.89	7.50	100	462.5	.....
1,314	8.06	8.30	100	478.5	.....
1,246	7.96	8.59	99	482.0	.....
1,145	7.75	8.23	99	441.2	.....
1,119	7.34	7.87	100	462.2	.....
1,145	6.76	8.55	100	432.4	.....
1,235	7.46	8.34	102	442.4	.....
1,434	8.69	8.99	101	511.9	.....
1,377	7.59	6.94	100	456.4	.....
1,514	8.00	6.54	99	456.3	.....
T 16,037	A 7.89	A 7.62	A 100	T 5,583.6	.....
1,857	9.37	3.99	98	517.3	.....
1,505	7.24	4.86	99	455.2	.....
1,464	7.85	5.79	98	493.4	.....
1,336	8.55	7.21	98	499.6	.....
1,221	7.24	8.59	97	466.7	.....
1,160	7.84	8.74	97	446.1	$12.70
1,411	8.18	8.77	97	448.3	12.54
1,272	4.58	9.68	101	383.7	12.53
1,615	4.63	8.78	102	414.9	12.48
1,686	5.61	8.62	97	451.9	12.26
1,815	5.40	7.35	97	420.7	12.32
1,938	6.53	5.15	97	450.9	12.56
T 18,280	A 6.92	A 7.29	A 98	T 5,448.7	A $12.48

See note on "Sources" pages 134, 135.

Year and Month	Unfilled Orders U. S. Steel Corp. (10,000 Long Tons) (1)	Gasoline Production in U. S. (1,000,000 Gallons) (2)	Pig Iron Production. Coke and Anthracite Pig Iron in the U. S. (1,000 Long Tons) (3)	Total Coke Production U. S. (1,000 Net Tons) (4)	Freight Car Loadings. Weekly Average (1,000 Cars) (5)
1915 Jan.	425	.....	1,601	2,304	.....
Feb.	435	.....	1,675	2,427	.....
Mar.	426	.....	2,064	2,836	.....
Apr.	416	.....	2,116	2,868	.....
May	426	.....	2,263	3,020	.....
June	468	.....	2,381	3,308	.....
July	493	.....	2,563	3,541	.....
Aug.	491	.....	2,780	3,834	.....
Sept.	532	.....	2,853	3,919	.....
Oct.	617	.....	3,125	4,450	.....
Nov.	719	.....	3,037	4,475	.....
Dec.	781	.....	3,203	4,599	.....
Total or Average	A 519	.....	A 2,472	T 41,581	.....
1916 Jan.	792	.....	3,185	4,425	.....
Feb.	857	.....	3,087	4,336	.....
Mar.	933	.....	3,338	4,827	.....
Apr.	983	.....	3,228	4,381	.....
May	994	.....	3,361	4,627	.....
June	964	.....	3,212	4,444	.....
July	959	.....	3,225	4,304	.....
Aug.	966	.....	3,204	4,620	.....
Sept.	952	.....	3,202	4,637	.....
Oct.	1,002	.....	3,509	4,795	.....
Nov.	1,106	.....	3,312	4,593	.....
Dec.	1,155	.....	3,179	4,544	.....
Total or Average	A 972	.....	A 3,253	T 54,533	.....

See note on "Sources" pages 134, 135.

Number of Business Failures, Dun's (6)	Bank Clearings New York City ($1,000,000,000) (7)	Portland Cement Production, U. S. (1,000,000 Barrels) (8)	Bureau of Labor Statistics Index Number of Wholesale Prices (1913 = 100) (9)	Domestic Cotton Consumption (1,000 Running Bales) (10)	Average Weekly Earnings in Representative New York State Factories (Dollars per Man) (11)
2,848	7.29	3.14	98	467.9	$ 12.44
2,278	6.48	3.50	99	463.3	12.41
2,090	7.57	5.80	99	524.9	12.65
2,063	8.81	7.79	99	514.0	12.54
1,707	8.64	8.92	100	493.8	12.74
1,754	8.03	8.85	99	514.7	12.81
1,739	8.70	8.18	100	496.8	12.66
1,395	8.54	8.46	100	464.4	12.89
1,414	9.62	8.11	100	498.7	12.86
1,599	12.74	8.53	102	500.8	13.30
1,565	11.83	8.01	104	514.7	13.45
1,704	12.33	6.47	108	555.0	13.49
T 22,156	A 9.22	A 7.15	A 101	T 6,009.0	A $12.85
2,009	12.33	4.74	113	542.1	$13.53
1,588	11.11	4.90	115	540.7	13.77
1,690	12.55	6.12	119	613.8	13.96
1,399	11.62	7.83	121	531.7	14.15
1,482	12.56	8.89	122	575.6	14.24
1,227	12.55	8.58	123	570.6	14.41
1,207	11.44	7.77	123	489.5	14.11
1,394	11.77	8.70	126	557.8	14.44
1,154	14.36	8.85	130	528.3	14.87
1,240	15.71	9.25	136	550.7	14.95
1,251	16.65	8.43	146	583.0	15.16
1,252	16.94	7.03	149	536.7	15.51
T 16,993	A 13.30	A 7.59	A 127	T6,620.0	A $14.43

See note on "Sources" pages 134, 135.

Year and Month	Unfilled Orders U. S. Steel Corp. (10,000 Long Tons) (1)	Gasoline Production in U. S. (1,000,000 Gallons) (2)	Pig Iron Production. Coke and Anthracite Pig Iron in the U. S. (1,000 Long Tons) (3)	Total Coke Production U. S. (1,000 Net Tons) (4)	Freight Car Loadings. Weekly Average (1,000 Cars) (5)
1917 Jan.	1,147	203.6	3,151	4,711	.....
Feb.	1,158	184.6	2,645	4,297	.....
Mar.	1,171	220.5	3,251	4,952	.....
Apr.	1,218	228.9	3,335	4,673	.....
May	1,189	238.8	3,417	4,740	.....
June	1,138	233.7	3,270	4,635	.....
July	1,084	244.1	3,342	4,636	.....
Aug.	1,041	254.5	3,248	4,565	.....
Sept.	988	256.1	3,134	4,646	.....
Oct.	901	271.9	3,303	4,678	.....
Nov.	890	264.9	3,206	4,577	930.9
Dec.	938	248.8	2,883	4,497	761.2
Total or Average	A 1,072	T 2,850.5	A 3,182	T 55,607	.....
1918 Jan.	948	242.6	2,412	3,894	652.3
Feb.	929	234.3	2,319	3,759	753.1
Mar.	906	269.6	3,213	4,680	848.7
Apr.	874	293.4	3,288	4,593	851.7
May	834	319.4	3,446	4,849	897.2
June	892	315.0	3,324	4,793	984.6
July	888	332.0	3,421	5,123	934.8
Aug.	876	330.3	3,390	5,016	962.3
Sept.	830	314.6	3,418	4,983	950.8
Oct.	835	314.3	3,487	5,168	934.3
Nov.	812	313.0	3,354	4,844	819.3
Dec.	738	291.7	3,434	4,777	751.0
Total or Average	A 864	T 3,570.3	A 3,209	T 56,479	A 867.0

See note on "Sources" pages 134, 135.

Number of Business Failures, Dun's (6)	Bank Clearings New York City ($1,000,000,000) (7)	Portland Cement Production, U. S. (1,000,000 Barrels) (8)	Bureau of Labor Statistics Index Number of Wholesale Prices (1913 = 100) (9)	Domestic Cotton Consumption (1,000 Running Bales) (10)	Average Weekly Earnings in Representative New York State Factories (Dollars per Man) (11)
1,540	15.13	6.15	153	601.4	$15.28
1,165	12.79	5.95	157	547.2	15.31
1,232	14.23	7.09	162	603.9	15.79
1,069	14.65	8.60	173	552.2	15.50
1,296	15.58	9.77	183	615.4	16.08
1,186	16.10	9.01	185	574.1	16.20
1,137	15.19	8.61	188	537.8	16.17
1,149	14.68	8.85	189	569.5	16.44
963	13.88	8.01	187	522.4	16.97
1,082	15.72	8.06	183	584.9	17.33
981	14.83	6.99	183	590.4	17.69
1,055	14.61	5.77	182	516.5	17.71
T 13,855	A 14.78	A 7.72	A 177	T 6,815.7	A $16.37
1,178	14.72	3.27	184	523.9	$16.81
980	12.26	3.70	186	510.1	17.66
1,142	13.84	4.68	187	571.4	18.71
905	14.09	6.37	190	544.1	19.25
880	15.56	7.70	190	575.9	19.91
804	14.86	7.93	191	515.8	20.44
786	15.40	7.60	196	541.5	20.78
720	14.96	7.47	200	535.0	21.23
674	13.66	6.70	204	490.0	22.31
660	16.92	5.93	202	440.4	22.34
570	15.61	5.08	203	455.6	21.60
683	16.66	4.26	202	472.9	23.18
T 9,982	A 14.88	A 5.90	A 194	T 6,176.6	A $20.35

See note on "Sources" pages 134, 135.

Year and Month	Unfilled Orders U. S. Steel Corp. (10,000 Long Tons) (1)	Gasoline Production in U. S. (1,000,000 Gallons) (2)	Pig Iron Production. Coke and Anthracite Pig Iron in the U. S. (1,000 Long Tons) (3)	Total Coke Production U. S. (1,000 Net Tons) (4)	Freight Car Loadings. Weekly Average (1,000 Cars) (5)
1919 Jan.	668	303.7	3,302	4,811	728.0
Feb.	601	283.5	2,940	3,920	686.8
Mar.	543	311.3	3,090	3,999	697.4
Apr.	480	319.8	2,478	3,302	714.7
May	428	354.5	2,108	3,007	758.6
June	489	338.3	2,115	3,078	809.4
July	558	342.5	2,429	3,701	858.0
Aug.	611	326.8	2,743	3,947	892.3
Sept.	628	339.6	2,488	3,904	960.0
Oct.	647	363.5	1,864	3,252	966.7
Nov.	713	338.7	2,392	3,600	807.2
Dec.	827	335.7	2,633	3,660	757.7
Total or Average	A 600	T 3,957.9	A 2,549	T 44,181	A 803.1
1920 Jan.	929	336.7	3,015	4,372	819.8
Feb.	950	322.6	2,979	4,048	776.2
Mar.	989	367.1	3,376	4,622	847.9
Apr.	1,036	355.6	2,740	3,846	730.5
May	1,094	381.1	2,986	4,071	861.6
June	1,098	415.2	3,044	4,170	859.9
July	1,112	423.4	3,067	4,324	901.0
Aug.	1,081	444.1	3,147	4,491	967.6
Sept.	1,037	453.9	3,129	4,475	968.7
Oct.	984	465.8	3,293	4,631	1,005.1
Nov.	902	452.6	2,935	4,284	883.7
Dec.	815	464.4	2,704	4,011	722.8
Total or Average	A 1,002	T 4,882.5	A 3,035	T 51,345	A 865.1

See note on "Sources" pages 134, 135.

Number of Business Failures, Dun's (6)	Bank Clearings New York City ($1,000,000,000) (7)	Portland Cement Production, U. S. (1,000,000 Barrels) (8)	Bureau of Labor Statistics Index Number of Wholesale Prices (1913 = 100) (9)	Domestic Cotton Consumption (1,000 Running Bales) (10)	Average Weekly Earnings in Representative New York State Factories (Dollars per Man) (11)
673	17.86	3.08	199	556.9	$23.03
602	14.19	3.52	193	433.3	22.07
629	16.49	4.56	196	433.5	22.20
543	17.33	6.25	199	475.9	22.11
531	18.88	7.57	202	487.9	22.23
485	19.75	7.72	203	474.3	22.51
452	21.88	8.26	212	510.3	23.10
468	19.53	8.68	216	497.3	23.85
473	19.61	8.98	210	491.1	24.83
463	23.71	9.05	211	556.0	24.41
551	22.59	7.83	217	491.3	25.37
581	23.98	4.92	223	511.7	26.32
T 6,451	A 19.65	A 6.70	A 206	5,919.5	A $23.50
569	23.21	6.29	233	591.9	$26.52
492	18.14	6.09	232	515.7	26.47
566	22.33	7.06	234	575.8	27.87
504	21.80	7.76	245	566.9	27.80
547	19.74	8.87	247	541.4	28.45
674	20.51	8.85	243	555.2	28.77
681	19.83	8.54	241	525.5	28.49
673	17.89	9.46	231	483.6	28.71
677	18.60	9.47	226	458.0	28.73
923	20.66	10.33	211	401.3	28.93
1,050	19.43	9.43	196	332.7	28.70
1,525	20.98	7.54	179	295.3	28.35
T 8,881	A 20.26	A 8.31	A 226	T 5,843.3	A $28.15

See note on "Sources" pages 134, 135.

Year and Month	Unfilled Orders U. S. Steel Corp. (10,000 Long Tons) (1)	Gasoline Production in U. S. (1,000,000 Gallons) (2)	Pig Iron Production. Coke and Anthracite Pig Iron in the U. S. (1,000 Long Tons) (3)	Total Coke Production U. S. (1,000 Net Tons) (4)	Freight Car Loadings. Weekly Average (1,000 Cars) (5)
1921 Jan.	757	460.4	2,416	3,347	704.8
Feb.	693	388.2	1,937	2,742	683.1
Mar.	628	419.8	1,596	2,335	692.2
Apr.	585	426.2	1,193	1,836	706.0
May	548	448.6	1,221	1,879	756.8
June	512	430.3	1,065	1,629	765.2
July	483	419.6	865	1,467	750.7
Aug.	453	431.6	954	1,621	809.9
Sept.	456	416.9	986	1,702	841.4
Oct.	429	441.0	1,247	2,138	929.2
Nov.	425	431.9	1,415	2,231	760.8
Dec.	427	339.0	1,649	2,361	682.9
Total or Average	A 533	T 5,153.5	A 1,379	T 25,288	A 756.9
1922 Jan.	424	444.6	1,645	2,415	702.2
Feb.	414	398.2	1,630	2,386	764.6
Mar.	449	472.3	2,036	2,918	825.8
Apr.	510	472.9	2,072	2,770	723.3
May	525	513.7	2,307	3,009	786.7
June	564	525.9	2,361	3,085	842.4
July	578	569.7	2,405	2,977	825.0
Aug.	595	550.0	1,816	2,389	876.5
Sept.	669	536.5	2,034	2,898	935.1
Oct.	690	566.3	2,638	3,747	992.5
Nov.	684	567.1	2,850	4,145	943.7
Dec.	675	585.0	3,087	4,385	838.4
Total or Average	A 565	T 6,202.2	A 2,240	T 37,124	A 838.0

See note on "Sources" pages 134, 135.

Number of Business Failures, Dun's (6)	Bank Clearings New York City ($1,000,000,000) (7)	Portland Cement Production, U. S. (1,000,000 Barrels) (8)	Bureau of Labor Statistics Index Number of Wholesale Prices (1913 = 100) (9)	Domestic Cotton Consumption (1,000 Running Bales) (10)	Average Weekly Earnings in Representative New York State Factories (Dollars per Man) (11)
1,895	18.57	4.10	170	366.5	$27.61
1,641	14.53	4.38	160	395.1	26.77
1,336	16.68	6.76	155	438.2	26.97
1,487	15.54	8.65	148	409.2	26.20
1,356	15.85	9.28	145	440.7	25.86
1,320	16.85	9.30	142	461.9	25.71
1,444	15.36	9.57	141	410.1	25.26
1,562	14.56	10.24	142	467.1	25.43
1,466	15.08	10.03	141	484.7	25.07
1,713	16.03	10.51	142	494.3	24.53
1,988	16.82	8.92	141	527.9	24.32
2,444	18.48	6.56	140	510.9	24.91
T 19,652	A 16.20	A 8.19	A 147	T 5,406.6	A $25.72
2,723	17.30	4.29	138	526.7	$24.43
2,331	15.34	4.28	141	472.3	24.17
2,463	18.72	6.69	142	519.8	24.57
2,167	18.76	9.24	143	443.5	24.15
1,960	19.22	11.18	148	495.3	24.59
1,740	20.11	11.25	150	509.2	24.91
1,753	18.34	11.56	155	458.0	24.77
1,714	16.94	11.66	155	526.4	25.10
1,556	17.29	11.42	153	494.0	25.71
1,708	19.67	12.29	154	533.7	25.61
1,737	19.33	11.35	156	579.2	26.04
1,814	18.90	8.67	156	529.3	26.39
T 23,676	A 18.16	A 9.49	A 149	T 6,087.4	A $25.04

See note on "Sources" pages 134, 135.

Year and Month	Unfilled Orders U. S. Steel Corp. (10,000 Long Tons) (1)	Gasoline Production in U. S. (1,000,000 Gallons) (2)	Pig Iron Production. Coke and Anthracite Pig Iron in the U. S. (1,000 Long Tons) (3)	Total Coke Production U. S. (1,000 Net Tons) (4)	Freight Car Loadings. Weekly Average (1,000 Cars (5)
1923 Jan.	691	623.7	3,230	4,577	845.1
Feb.	728	568.7	2,994	4,291	841.7
Mar.	740	630.7	3,524	5,004	916.6
Apr.	729	619.0	3,550	4,980	941.0
May	698	631.7	3,868	5,155	974.7
June	639	636.7	3,676	4,919	1,011.3
July	591	636.9	3,678	4,848	986.1
Aug.	542	649.0	3,449	4,732	1,040.9
Sept.	504	623.7	3,126	4,485	1,036.8
Oct.	467	659.1	3,149	4,390	1,077.7
Nov.	437	617.6	2,894	4,044	977.8
Dec.	445	659.2	2,921	4,062	826.5
Total or Average	A 601	T 7,555.9	A 3,338	T 55,487	A 956.4
1924 Jan.	480	695.3	3,019	4,248	908.1
Feb.	491	683.7	3,075	4,192	915.7
Mar.	478	743.2	3,466	4,563	874.8
Apr.	421	754.8	3,233	4,089	894.9
May	363	779.2	2,615	3,559	858.4
June	326	737.1	2,026	2,971	906.4
July	319	742.0	1,785	2,833	894.4
Aug.	329	755.8	1,887	2,879	974.4
Sept.	347	750.3	2,053	3,066	1,036.6
Oct.	353	760.6	2,477	3,530	1,090.7
Nov.	403	762.0	2,510	3,559	974.7
Dec.	482	795.6	2,962	3,967	847.5
Total or Average	A 399	T 8,959.6	A 2,592	T 43,456	A 931.4

See note on "Sources" pages 134, 135.

Number of Business Failures, Dun's (6)	Bank Clearings New York City ($1,000,000,000) (7)	Portland Cement Production, U. S. (1,000,000 Barrels) (8)	Bureau of Labor Statistics Index Number of Wholesale Prices (1913=100) (9)	Domestic Cotton Consumption (1,000 Running Bales) (10)	Average Weekly Earnings in Representative New York State Factories (Dollars per Man) (11)
2,126	19.78	8.00	156	610.3	$26.21
1,508	16.78	8.21	157	566.8	25.87
1,682	19.77	9.88	159	624.3	26.92
1,520	18.01	11.36	159	576.5	27.00
1,530	19.21	12.91	156	621.0	27.63
1,358	18.68	12.38	153	542.0	27.87
1,231	16.65	12.62	151	461.6	27.54
1,319	14.78	12.97	150	493.0	27.12
1,226	15.07	13.11	154	483.9	27.41
1,673	17.73	13.35	153	543.7	27.72
1,704	18.05	12.60	152	531.6	27.64
1,841	19.50	10.00	151	464.2	27.98
T 18,718	A 17.83	A 11.45	A 154	T 6,518.9	A $27.24
2,108	20.69	8.79	151	579.8	$27.81
1,730	18.12	8.59	152	508.7	27.73
1,817	19.65	10.37	150	483.9	28.16
1,707	20.33	11.73	148	481.6	27.70
1,816	20.72	13.78	147	413.6	27.56
1,607	19.96	13.54	145	350.3	27.21
1,615	21.13	14.03	147	346.7	27.06
1,520	20.34	15.13	150	357.5	27.40
1,306	19.29	14.52	149	435.2	28.05
1,696	21.58	14.82	152	532.6	27.53
1,653	22.43	13.14	153	492.2	27.66
2,040	25.63	10.43	157	532.0	28.25
T 20,615	A 20.80	A 12.40	A 150	T 5,514.1	A $27.68

See note on "Sources" pages 134, 135.

Year and Month	Unfilled Orders U. S. Steel Corp. (10,000 Long Tons) (1)	Gasoline Production in U. S. (1,000,000 Gallons) (2)	Pig Iron Production. Coke and Anthracite Pig Iron in the U. S. (1,000 Long Tons) (3)	Total Coke Production U. S. (1,000 Net Tons) (4)	Freight Car Loadings. Weekly Average (1,000 Cars) (5)
1925 Jan.	504	831.7	3,370	4,581	921.3
Feb.	528	790.4	3,214	4,179	904.8
Mar.	486	853.6	3,564	4,462	923.7
Apr.	445	860.5	3,259	4,122	940.7
May	405	922.0	2,931	3,903	968.3
June	371	944.2	2,673	.....	989.0
July	354	.....	2,664	.....	986.2
Aug.	.....	.....	.....	.....	.....
Sept.	.....	.....	.....	.....	.....
Oct.	.....	.....	.....	.....	.....
Nov.	.....	.....	.....	.....	.....
Dec.	.....	.....	.....	.....	.....
Total or Average	.....	.....	.....	.....	.....
1926 Jan.	.....	.....	.....	.....	.....
Feb.	.....	.....	.....	.....	.....
Mar.	.....	.....	.....	.....	.....
Apr.	.....	.....	.....	.....	.....
May	.....	.....	.....	.....	.....
June	.....	.....	.....	.....	.....
July	.....	.....	.....	.....	.....
Aug.	.....	.....	.....	.....	.....
Sept.	.....	.....	.....	.....	.....
Oct.	.....	.....	.....	.....	.....
Nov.	.....	.....	.....	.....	.....
Dec.	.....	.....	.....	.....	.....
Total or Average	.....	.....	.....	.....	.....

See note on "Sources" pages 134, 135.

Number of Business Failures, Dun's (6)	Bank Clearings New York City ($1,000,000,000) (7)	Portland Cement Production, U. S. (1,000,000 Barrels) (8)	Bureau of Labor Statistics Index Number of Wholesale Prices (1913 = 100) (9)	Domestic Cotton Consumption (1,000 Running Bales) (10)	Average Weekly Earnings in Representative New York State Factories (Dollars per Man) (11)
2,317	26.72	8.86	160	589.7	$28.30
1,793	21.06	8.26	161	550.1	27.96
1,859	23.35	11.03	161	582.7	28.45
1,939	22.85	13.81	156	597.1	27.67
1,767	23.85	15.50	155	531.5	28.07
1,745	24.02	15.39	157	493.8	27.94
1,685	23.40	15.64	160	483.9	.....
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....

Year and Month	Unfilled Orders U. S. Steel Corp. (10,000 Long Tons) (1)	Gasoline Production in U. S. (1,000,000 Gallons) (2)	Pig Iron Production. Coke and Anthracite Pig Iron in the U. S. (1,000 Long Tons) (3)	Total Coke Production U. S. (1,000 Net Tons) (4)	Freight Car Loadings. Weekly Average (1,000 Cars) (5)
1927 Jan.					
Feb.					
Mar.					
Apr.					
May					
June					
July					
Aug.					
Sept.					
Oct.					
Nov.					
Dec.					
Total or Average					

SOURCES:

(1) *The Iron Trade Review.*

(2) U. S. Department of the Interior, Bureau of Mines. Standard Daily Trade Service.

(3) *The Iron Age.*

(4) U. S. Geological Survey. *Survey of Current Business.*

(5) American Railway Association and Standard Daily Trade Service.

Number of Business Failures, Dun's (6)	Bank Clearings New York City ($1,000,000,000) (7)	Portland Cement Production, U. S. (1,000,000 Barrels) (8)	Bureau of Labor Statistics Index Number of Wholesale Prices (1913=100) (9)	Domestic Cotton Consumption (1,000 Running Bales) (10)	Average Weekly Earnings in Representative New York State Factories (Dollars per Man) (11)

(6) *Dun's Review.*
(7) *Commercial and Financial Chronicle.*
(8) U. S. Geological Survey. Standard Daily Trade Service.
(9) Bureau of Labor Statistics.
(10) U. S. Bureau of the Census. Standard Daily Trade Service.
(11) *The Industrial Bulletin.* New York State Department of Labor.

V

STATISTICAL FORMULAS

The page references given below are to the sections of the *Text* (or, in a few cases, of the *Manual*) in which the given formulas are explained.

A. Formulas Relating to the Frequency Distribution

	STATISTICAL MEASURE	FORMULA	PAGES OF TEXT
1.	Arithmetic Mean	$M = \frac{\Sigma(fm)}{N}$	113–115
2.	Arithmetic Mean	$M = M' + c$	116–119
3.	Correction to Assumed Mean	$c = \Sigma\left(\frac{fd'}{N}\right)$	116–119
4.	Median[1]	$Md = l + \left(\frac{\frac{N}{2} - \Sigma f_L}{f_M} \times i\right)$	122–123
5.	Mode (approximate)	$Mo = l + \left(\frac{f_2}{f_2 + f_1} \times i\right)$	127–129
6.	Mode (approximate, in a moderately skewed distribution)	$Mo = \text{Mean} - 3\,(\text{Mean} - Md)$	130
7.	Geometric Mean (unweighted)	$M_g = \sqrt[N]{a_1 \cdot a_2 \cdot a_3 \ldots a_n}$	135
8.	Logarithm of Geometric Mean (unweighted)	$\text{Log } M_g = \frac{\log a_1 + \log a_2 + \log a_3 + \ldots + \log a_n}{N}$	135
9.	Geometric Mean (weighted)	$M_g = \sqrt[N]{a_1^{w_1} \cdot a_2^{w_2} \cdot a_3^{w_3} \ldots a_n^{w_n}}$	135
10.	Logarithm of Geometric Mean (weighted)	$\text{Log } M_g = \frac{w_1 \log a_1 + w_2 \log a_2 + w_3 \log a_3 + \ldots + w_n \log a_n}{N}$	136

[1] This formula is not given in the *Text*, but may serve as a useful summary of the operations involved in locating the median. The symbol f_L is used to represent the frequencies in all classes below the class containing the median, f_M is the frequency in the median class, l is the lower limit of the class containing the median and i is the class-interval. Similar formulas may be applied in locating quartiles and deciles.

STATISTICAL MEASURE	FORMULA	PAGES OF TEXT
11. Harmonic Mean	$\frac{1}{H} = \frac{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots + \frac{1}{r_n}}{N}$	141–142
12. Harmonic Mean (two items)	$H = \frac{2ab}{a + b}$	142
13. Harmonic Mean (three items)	$H = \frac{3abc}{ab + ac + bc}$	142
14. Mean Deviation	$M.D. = \frac{\Sigma d}{N}$	149–150
15. Mean Deviation	$M.D. = \frac{\Sigma(fd') + (N_s - N_l)c}{N}$	151–154
16. Mean Deviation from Mean	$M.D. = \frac{2(FM - \Sigma m_L)}{N}$	*Manual* 16
17. Standard Deviation	$\sigma = \sqrt{\frac{\Sigma fd^2}{N}}$	154–155
18. Standard Deviation	$\sigma^2 = \frac{\Sigma f(d')^2}{N} - c^2$	155–157
19. Standard Deviation	$\sigma^2 = s^2 - c^2$	155–157
20. Standard Deviation (after Sheppard's correction applied)	$\sigma = \sqrt{\mu_2}$	531
21. Charlier check upon calculation of Standard Deviation	$\Sigma f(d'+1)^2 = \Sigma f(d')^2 + 2\Sigma fd' + N$	156–157
22. Quartile Deviation	$Q.D. = \frac{Q_3 - Q_1}{2}$	158–159
23. Probable Error	$P.E. = 0.6745\,\sigma$	160–161
24. Coefficient of Variation	$V = \frac{\sigma}{M} \times 100$	165
25. Measure of Skewness	$sk = \frac{M - Mo}{\sigma}$	166–167
26. Measure of Skewness	$sk = \frac{q_2 - q_1}{q_2 + q_1}$	167
27. Measure of Skewness	$\chi = \frac{\sqrt{\beta_1}(\beta_2 + 3)}{2(5\beta_2 - 6\beta_1 - 9)}$	546
28. Mean number of successes where probabilities and number of independent events are known	$M = np$	525

STATISTICAL MEASURE	FORMULA	PAGES OF TEXT
29. Standard Deviation, same conditions	$\sigma = \sqrt{npq}$	526
30. Equation to Normal Curve of Error	$y = y_0 e^{-\frac{x^2}{2\sigma^2}}$	527
31. Equation to Normal Curve of Error	$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$	527
32. Maximum ordinate, Normal Curve	$y_0 = \frac{N}{\sigma\sqrt{2\pi}}$	527
33. Moments of a frequency distribution about arbitrary origin		
First moment	$\nu_1 = \frac{\Sigma f(x')}{N}$	529
Second moment	$\nu_2 = \frac{\Sigma f(x')^2}{N}$	
Third moment	$\nu_3 = \frac{\Sigma f(x')^3}{N}$	
Fourth moment	$\nu_4 = \frac{\Sigma f(x')^4}{N}$	
34. Moments of a frequency distribution about the Mean		
First moment	$\pi_1 = 0$	529
Second moment	$\pi_2 = \nu_2 - \nu_1^2$	
Third moment	$\pi_3 = \nu_3 - 3\nu_1\nu_2 + 2\nu_1^3$	
Fourth moment	$\pi_4 = \nu_4 - 4\nu_1\nu_3 + 6\nu_1^2\nu_2 - 3\nu_1^4$	
35. Corrected moments of a frequency distribution about the Mean		
First moment	$\mu_1 = 0$	530
Second moment	$\mu_2 = \pi_2 - \frac{1}{12}$	
Third moment	$\mu_3 = \pi_3$	
Fourth moment	$\mu_4 = \pi_4 - \frac{1}{2}\pi_2 + \frac{7}{240}$	
36. Criteria of curve type	$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$	532
	$\beta_2 = \frac{\mu_4}{\mu_2^2}$	
	$\kappa_2 = \frac{\beta_1(\beta_2+3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}$	

STATISTICAL MEASURE	FORMULA	PAGES OF TEXT
37. Standard error of sampling	$\sigma_s = \sqrt{\frac{f(N-f)}{N}}$	541
38. Chi-square, for testing goodness of fit	$\chi^2 = \Sigma\left(\frac{(f_0 - f)^2}{f}\right)$	544
39. Measure of Kurtosis	$\beta_2 - 3$	545
40. Distance, d, between Mean and Mode (where χ is measure of skewness given in formula 27)	$d = \chi \times \sigma$	546

B. Formulas Relating to Index Numbers

41. Simple aggregative index number of prices	$\frac{P_1}{P_0} = \frac{\Sigma p_1}{\Sigma p_0}$	193–194
42. Simple arithmetic average of relative prices	$M = \frac{\Sigma\left(\frac{p_1}{p_0}\right)}{N}$	195–196
43. Geometric average of relative prices	$M_g = \sqrt[N]{\frac{p_1'}{p_0'} \times \frac{p_1''}{p_0''} \times \frac{p_1'''}{p_0'''} \times \ldots}$	198
44. Logarithm of geometric average of relative prices	$\text{Log } M_g = \frac{\log\left(\frac{p_1'}{p_0'}\right) + \log\left(\frac{p_1''}{p_0''}\right) + \log\left(\frac{p_1'''}{p_0'''}\right) + \ldots}{N}$	198–199
45. Harmonic average of relative prices	$H = \frac{N}{\Sigma\left(\frac{p_0}{p_1}\right)}$	199–200
46. Weighted aggregative index number of prices (base year weights)	$\frac{\Sigma\, p_1 q_0}{\Sigma\, p_0 q_0}$	207–208
47. Weighted aggregative index number of prices (given year weights)	$\frac{\Sigma\, p_1 q_1}{\Sigma\, p_0 q_1}$	207–208
48. "Ideal" index	$\sqrt{\frac{\Sigma\, p_1 q_0}{\Sigma\, p_0 q_0} \times \frac{\Sigma\, p_1 q_1}{\Sigma\, p_0 q_1}}$	215–216
49. "Best practical substitute for ideal index"	$\frac{\Sigma\,(q_0 + q_1)p_1}{\Sigma\,(q_0 + q_1)p_0}$	218
50. Ideal index applied to measurement of changes in quantities	$\sqrt{\frac{\Sigma(q_1 p_0)}{\Sigma(q_0 p_0)} \times \frac{\Sigma(q_0 p_1)}{\Sigma(q_1 p_1)}}$	346–347

C. Formulas Relating to Time Series

	STATISTICAL MEASURE	FORMULA	PAGES OF TEXT
51.	Normal equations for fitting straight line (equation $y = a + bx$) by least squares	$\Sigma(y) = na + b\Sigma(x)$ $\Sigma(xy) = a\Sigma(x) + b\Sigma(x^2)$	275–277
52.	Normal equations for fitting second degree parabola (equation $y = a + bx + cx^2$) by least squares	$\Sigma(y) = na + b\Sigma(x) + c\Sigma(x^2)$ $\Sigma(xy) = a\Sigma(x) + b\Sigma(x^2) + c\Sigma(x^3)$ $\Sigma(x^2y) = a\Sigma(x^2) + b\Sigma(x^3) + c\Sigma(x^4)$	280–281
53.	Normal equations for fitting curve of type $\log y = a + bx + cx^2$ by least squares	$\Sigma(\log y) = na + b\Sigma(x) + c\Sigma(x^2)$ $\Sigma(x \cdot \log y) = a\Sigma(x) + b\Sigma(x^2) + c\Sigma(x^3)$ $\Sigma(x^2 \cdot \log y) = a\Sigma(x^2) + b\Sigma x^3 + c\Sigma(x^4)$	290–291

(See Formula 96 for a generalized set of normal equations.)

54.	Equation to Gompertz curve	$y = ab^{c^x}$	297
55.	Equation to Pearl-Reed curve of population growth	$y = d + \frac{b}{e^{-ax} + c}$	297
56.	Interest rate at which a sum is compounded	$r = \sqrt[n]{\frac{p_n}{p_o}} - 1$	138

(See *Text* p. 320 for application of above formula to link relative method of measuring seasonal variation.)

D. Formulas Relating to the Measurement of Correlation

57.	General formula for a measure of correlation	$\sqrt{1 - \frac{S_y^2}{\sigma_y^2}}$	373
58.	Coefficient of correlation (where S_y^2 is the mean-square deviation about a straight line)	$r = \sqrt{1 - \frac{S_y^2}{\sigma_y^2}}$	373–374
59.	Coefficient of correlation	$r^2 = \frac{a\Sigma(Y) + b\Sigma(XY) - Nc_y^2}{\Sigma(Y^2) - Nc_y^2}$	377
60.	Special formula for r when origin is at mean of X's	$r^2 = \frac{b\Sigma(xY)}{\Sigma(Y^2) - Nc_y^2}$	572

STATISTICAL MEASURE	FORMULA	PAGES OF TEXT
61. Special formula for r when origin is at mean of Y's	$r^2 = \frac{b\Sigma(Xy)}{\Sigma(y^2)}$	573
62. Coefficient of correlation	$r = \frac{\Sigma(xy)}{N\sigma_x\sigma_y}$	386
63. Coefficient of correlation	$r = \frac{p}{\sigma_x\sigma_y}$	403
64. Mean product	$p = \frac{\Sigma(XY)}{N} - c_x c_y$	403
65. Corrections in above formula	$c_x = \frac{\Sigma(X)}{N}$; $c_y = \frac{\Sigma(Y)}{N}$	403
66. Standard deviations	$\sigma_x = \sqrt{\frac{\Sigma(X^2)}{N} - c_x^2}$; $\sigma_y = \sqrt{\frac{\Sigma(Y^2)}{N} - c_y^2}$	403
67. Coefficient of correlation	$r = \sqrt{b_{yx} \cdot b_{xy}}$	398
68. Coefficient of regression	$b_{yx} = r\frac{\sigma_y}{\sigma_x}$	398
69. Coefficient of regression	$b_{xy} = r\frac{\sigma_x}{\sigma_y}$	398
70. Equation to line of regression (y dependent)	$y = r\frac{\sigma_y}{\sigma_x}x$	393
71. Equation to line of regression (x dependent)	$x = r\frac{\sigma_x}{\sigma_y}y$	397
72. Equation to line of regression (Y dependent)	$Y - \overline{Y} = r\frac{\sigma_y}{\sigma_x}(X - \overline{X})$	403
73. Equation to line of regression (X dependent)	$X - \overline{X} = r\frac{\sigma_x}{\sigma_y}(Y - \overline{Y})$	403
74. General formula for standard error of estimate	$S_y^2 = \frac{\Sigma(d^2)}{N}$	376
75. Standard error of estimate (straight line)	$S_y^2 = \frac{\Sigma(Y^2) - a\Sigma(Y) - b\Sigma(XY)}{N}$	377
76. Standard errors of estimate	$S_y = \sigma_y\sqrt{1 - r^2}$; $S_x = \sigma_x\sqrt{1 - r^2}$	403
77. General formula for index of correlation (Y dependent)	$\rho^2_{yx} = 1 - \frac{S_y^2}{\sigma_y^2}$	436–437
78. General formula for index of correlation (X dependent)	$\rho^2_{xy} = 1 - \frac{S_x^2}{\sigma_x^2}$	437

STATISTICAL MEASURE	FORMULA	PAGES OF TEXT
79. Index of correlation (curve of potential series)	$\rho^2_{yx} = \dfrac{a\Sigma(Y) + b\Sigma(XY) + c\Sigma(X^2Y) + d\Sigma(X^3Y) + \ldots - Nc_y^2}{\Sigma(Y^2) - Nc_y^2}$	439
80. Index of correlation (curve of potential series; origin at mean of Y's)	$\rho^2_{yx} = \dfrac{b\Sigma(Xy) + c\Sigma(X^2y) + d\Sigma(X^3y) + \ldots}{\Sigma(y^2)}$	439
81. Standard error of estimate (curve of potential series)	$S_y^2 = \dfrac{\Sigma(Y^2) - a\Sigma(Y) - b\Sigma(XY) - c\Sigma(X^2Y) - d\Sigma(X^3Y) - \ldots}{N}$	439
82. Correlation ratio (Y dependent)	$\eta_{yx} = \sqrt{1 - \dfrac{\sigma_{ay}^2}{\sigma_y^2}}$	442
83. Correlation ratio (Y dependent)	$\eta_{yx} = \dfrac{\sigma_{my}}{\sigma_y}$	446
84. Correlation ratio (X dependent)	$\eta_{xy} = \dfrac{\sigma_{mx}}{\sigma_x}$	448
85. Square of corrected correlation ratio (where κ represents the number of arrays)	$= \dfrac{\eta^2 - \dfrac{(\kappa - 3)}{N}}{1 - \dfrac{(\kappa - 3)}{N}}$	450
86. Test for linearity	$\zeta = \eta^2 - r^2$	451
87. Coefficient of multiple correlation	$R^2_{1\cdot 234\ldots n} = 1 - \dfrac{S^2_{1\cdot 234\ldots n}}{\sigma_1^2}$	497
88. Coefficient of multiple correlation	$\dfrac{a\Sigma(X_1) + b_{12.34\ldots n}\Sigma(X_1X_2) + b_{13.24\ldots n}\Sigma(X_1X_3) + b_{14.23\ldots n}\Sigma(X_1X_4) + \ldots - Nc_1^2}{\Sigma(X_1)^2 - Nc_1^2}$	497
89. Coefficient of multiple correlation	$R^2_{1\cdot 234} = \dfrac{b_{12\cdot 34}p_{12} + b_{13\cdot 24}p_{13} + b_{14\cdot 23}p_{14}}{\sigma_1^2}$	497
90. Coefficient of multiple correlation	$1 - R^2_{1\cdot 23\ldots n} = (1 - r^2_{12})(1 - r^2_{13\cdot 2})(1 - r^2_{14\cdot 23})\ldots (1 - r^2_{1n\cdot 23\ldots (n-1)})$	514

STATISTICAL MEASURE	FORMULA	PAGES OF TEXT

91. Standard error of estimate (several independent variables)

$$S^2_{1.234\ldots n} = \frac{\Sigma(X_1^2) - a\Sigma(X_1) - b_{12.34\ldots n}\Sigma(X_1X_2) - b_{13.24\ldots n}\Sigma(X_1X_3) - b_{14.23\ldots n}\Sigma(X_1X_4) - \ldots}{N}$$ 496

92. Standard error of estimate (several independent variables)

$$S^2_{1.234} = \sigma_1^2 - b_{12.34}p_{12} - b_{13.24}p_{13} - b_{14.23}p_{14}$$ 495

93. Standard deviation of order n (standard error of estimate

$$\sigma^2_{1.23\ldots n} = \sigma_1^2(1 - r^2_{12})\ (1 - r^2_{13.2})\ (1 - r^2_{14.23})\ldots (1 - r^2_{1n.23\ldots n-1})$$ 513

94. Coefficient of partial correlation

$$r_{12.3456\ \ldots\ n} = \sqrt{b_{12.3456\ \ldots\ n} \cdot b_{21.3456\ \ldots\ n}}$$ 505

95. Coefficient of partial correlation

$$r_{12.345\ \ldots\ n} = \frac{r_{12.345\ \ldots\ (n-1)} - r_{1n.345\ \ldots\ (n-1)} \cdot r_{2n.345\ \ldots\ (n-1)}}{(1 - r^2_{1n.345\ \ldots\ (n-1)})^{\frac{1}{2}}\ (1 - r^2_{2n.345\ \ldots\ (n-1)})^{\frac{1}{2}}}$$ 508

96. A generalized set of normal equations, to be used in the fitting of any curve the equation to which can be put in the form $Y = aW_1 + bW_2 + cW_3 + dW_4 + \ldots$

I. $\Sigma(W_1Y) = a\Sigma(W_1^2) + b\Sigma(W_1W_2) + c\Sigma(W_1W_3) + d\Sigma(W_1W_4) + \ldots$ 566

II. $\Sigma(W_2Y) = a\Sigma(W_1W_2) + b\Sigma(W_2^2) + c\Sigma(W_2W_3) + d\Sigma(W_2W_4) + \ldots$

III. $\Sigma(W_3Y) = \Sigma(W_1W_3) + b\Sigma(W_2W_3) + c\Sigma(W_3^2) + d\Sigma(W_3W_4) + \ldots$

IV. $\Sigma(W_4Y) = a\Sigma(W_1W_4) + b\Sigma(W_2W_4) + c\Sigma(W_3W_4) + d\Sigma(W_4^2) + \ldots$

97. Generalized formula for sum of squares of residuals, for equation of above type

$$\Sigma(v^2) = \Sigma(Y)^2 - a\Sigma(W_iY) - b\Sigma(W_2Y) - c\Sigma(W_3Y) - d\Sigma(W_4Y) - \ldots$$ 569

STATISTICAL MEASURE	FORMULA	PAGES OF TEXT
98. Generalized formula for standard error of estimate, for equation of above type	$S_y^2 = \frac{\Sigma(Y^2) - a\Sigma(W_1Y) - b\Sigma(W_2Y) - c\Sigma(W_3Y) - d\Sigma(W_4Y)}{N}$	570
99. Generalized formula for index of correlation, for equation of above type	$\rho^2_{y \cdot w_1 w_2 w_3 w_4 \ldots} = \frac{a\Sigma(W_1Y) + b\Sigma(W_2Y) + c\Sigma(W_3Y) + d\Sigma(W_4Y) + \ldots - Nc_y^2}{\Sigma(Y^2) - Nc_y^2}$	571

E. Standard Errors and Probable Errors of Certain Statistical Measures

(Note: Where the formula for the *probable error* is not given, this measure may be derived by multiplying the value of the standard error by .6745.)

STATISTICAL MEASURE	MEASURE OF RELIABILITY
Arithmetic Mean	$\sigma_M = \frac{\sigma}{\sqrt{N}}$ $P.E._M = .6745\frac{\sigma}{\sqrt{N}}$
Median (normal distribution)	$\sigma_{Md} = 1.25331\frac{\sigma}{\sqrt{N}}$ $P.E._{Md} = .84535\frac{\sigma}{\sqrt{N}}$
Quartile (normal distribution)	$\sigma_{Q_1} = 1.36263\frac{\sigma}{\sqrt{N}}$ $P.E._{Q_1} = .91908\frac{\sigma}{\sqrt{N}}$

(These measures have the same values for Q_3 as for Q_1).

Standard deviation of a normal distribution	$\sigma_\sigma = \frac{\sigma}{\sqrt{2N}}$
Standard deviation, any type of distribution	$\sigma_\sigma = \sqrt{\frac{\mu_4 - \mu_2^2}{4\mu_2 \cdot N}}$
Coefficient of Variation (normal distribution)	$P.E._V = .4769\frac{V}{\sqrt{N}}\left[1+2\left(\frac{V}{100}\right)^2\right]^{\frac{1}{2}}$

STATISTICAL MEASURE	MEASURE OF RELIABILITY
Quartile deviation (normal distribution)	$P.E._{QD} = .5306\frac{\sigma}{\sqrt{N}}$
Second moment about Mean (normal distribution)	$P.E._{\mu_2} = .6745\ \sigma^2\sqrt{\frac{2}{N}}$
Third moment about Mean (normal distribution)	$P.E._{\mu_3} = .6745\ \sigma^3\sqrt{\frac{6}{N}}$
Fourth moment about Mean (normal distribution)	$P.E._{\mu_4} = .6745\ \sigma^4\sqrt{\frac{96}{N}}$
Difference between observed and computed frequencies (standard error of sampling)	$\sigma_s = \sqrt{\frac{f(N-f)}{N}}$
Coefficient of correlation (normal distribution)	$\sigma_r = \frac{1-r^2}{\sqrt{N}}$

(This formula holds also for coefficients of multiple and partial correlation).

Correlation ratio (This formula holds only approximately)	$\sigma_\eta = \frac{1-\eta^2}{\sqrt{N}}$
Coefficient of regression	$\sigma_{b12} = \frac{\sigma_1\sqrt{1-r^2}}{\sigma_2\sqrt{N}}$
Test for linearity	$\sigma_\zeta = 2\sqrt{\frac{\zeta}{N}}\sqrt{(1-\eta^2)^2-(1-r^2)^2+1}$
A value of Y, as computed from an equation of regression	$S_y = \sigma_y\sqrt{1-r^2}$
A value of X, as computed from an equation of regression	$S_x = \sigma_x\sqrt{1-r^2}$

(The above two equations may be modified by substituting ρ^2 for r^2 when the estimating equation is non-linear, or R^2 for r^2 when several independent variables are included.)

Difference between two Means	$\sigma_D = \sqrt{\frac{\sigma_1^2}{N_1}+\frac{\sigma_2^2}{N_2}}$

Criterion β_1 Criterion β_2 Criterion κ_2 Measure of skewness Distance between Mean and Mode	See Pearson, *Tables for Statisticians and Biometricians*, pages lxii to lxix, and Tables XXXVII, XXXVIII, XL, XLI and XLIII.

Certain of these formulas are discussed in the *Text*, Chapter XVI. For a more detailed treatment see the following references:

Jones, D. C. *A First Course in Statistics*, Chapters XII, XIII and XIV.

Kelley, Truman L. *Statistical Method*, Chapters IV and V.

Pearson, Karl. "On the Probable Errors of Frequency Constants," *Biometrika* vol. 2, 273, 281.

Rietz, H. L. "Random Sampling." (In: Rietz, H. L. *ed. Handbook of Mathematical Statistics.*)

Yule, G. U. An Introduction to the Theory of Statistics. 254–290, 335–356.

VI

GLOSSARY OF SYMBOLS

The following are the most important symbols employed in the *Text* and in the *Manual*. A given symbol is sometimes called upon to serve different purposes, but the precise meaning should be clear from the context.

1. General symbols for variables and constants:

x: a variable quantity.

y: a variable quantity.

In general, any letter near the end of the alphabet may be employed to represent a variable quantity. Different variable quantities may be represented by the use of a single symbol, with different subscripts, as X_1, X_2, X_3, or W_1, W_2, W_3. [A distinction is later drawn (*cf.* Symbols employed in the measurement of relationship) between capital letters and small letters, as used to represent variable quantities.]

a: a constant (i.e., a quantity the value of which does not change in the given discussion). In general, any letter near the beginning of the alphabet may be used to represent a constant.

2. Symbols employed in the analysis and description of the frequency distribution:

m: the value of an individual observation; the value of the midpoint of a class. (The symbols a_1, a_2, a_3 are sometimes employed to represent different observations in a series.)

f: the number of observations in a given class; frequency of a given class.

i: the class-interval.

l: the lower limit of a class.

N: the total number of cases in a given series or frequency distribution.

d: the deviation of a given observation from an average; usually, a deviation from the arithmetic mean. When written with a subscript, as d_x or d_y, it refers to a deviation from the arithmetic mean of the variable represented by the subscript. The symbol d is sometimes used to designate the difference between mean and mode.

d′: the deviation of a given observation from an arbitrary origin, or assumed mean.

c: the difference between an arbitrary origin, or assumed mean, and the true mean (in terms of the symbols explained below, $c = M - M'$).

Σ(Sigma): the symbol for the process of summation. Thus Σd means the sum of all the deviations.

w_1, w_2, w_3: weights attached to a series of measures being averaged. (Not to be confused with similar symbols used to represent different variable quantities.)

y_o: the maximum ordinate of a frequency curve.

Symbols for averages, quartiles, etc.:

M: the arithmetic mean.

$Md.$: the median.

$Mo.$: the mode.

M_g: the geometric mean.

H: the harmonic mean.

M^1: the value of an assumed arithmetic mean.

Q_1: the first or lower quartile.

Q_2: the second quartile or median.

Q_3: the third or upper quartile.

K: the value of a point midway between the first and third quartiles.

D_3: the third decile.

Symbols for measures of variation and skewness:

$M.D.$: the mean deviation.

σ: the standard deviation; the root-mean-square deviation about the arithmetic mean.

s: the root-mean-square deviation about an origin other than the arithmetic mean.

$P.E.$: the probable error.

$Q.D.$: the quartile deviation.

q_1: the difference between the median and the lower quartile ($Md. - Q_1$).

q_2: the difference between the upper quartile and the median ($Q_3 - Md.$).

V: the coefficient of variation.

sk: a measure of skewness.

χ(Chi): a measure of skewness based upon the criteria β_1 and β_2. (These criteria are explained below.)

3. Symbols relating to index numbers.

p_o': price of a given commodity at time "0" (the base period).

q_o': quantity of same commodity at time "0".

p_1': price of same commodity at time "1".

q_1': quantity of same commodity at time "1".

p_o'': price of a second commodity at time "0".
q_o'': quantity of second commodity at time "0".
p_1'': price of second commodity at time "1".
q_1'': quantity of second commodity at time "1".
$\frac{p_1'}{p_o'}$: a price relative (relation of price of a given commodity at time "1" to price of same commodity at time "0".)
$\frac{q_1'}{q_o'}$: a quantity relative.
P_o: price level at time "0"
P_1: price level at time "1".

4. Symbols employed in the measurement of relationship.

X: an observed value of a variable quantity.
Y: an observed value of a variable quantity. (The observed values of different variables may be represented also by the symbols X_1, X_2, X_3, or W_1, W_2, W_3.)
$\overline{X}$: the arithmetic mean of a number of observed values of the variable X. Similar symbols may be employed for other variables. (In one demonstration in the *Text* pp. 485–495, relating to multiple correlation, the symbols A_1, A_2, A_3 . . . are used to represent the arithmetic means of the variables X_1, X_2, X_3 . . . The symbols M_x and M_y are occasionally employed to designate the arithmetic means of different variables.)
x: value of a variable quantity expressed as a deviation from the arithmetic mean of all the observed values. The symbol y and the symbols x_1, x_2, x_3 . . . are similarly employed with respect to variables represented, as to original observations, by the symbols Y, X_1, X_2, X_3 . . .
x': a value of a variable quantity expressed as a deviation from an arbitrary origin. The symbol y' has a similar meaning.
Y_c: the computed or estimated value of a variable, as determined from an equation of average relationship; the symbol y_c may be employed for such a computed value, expressed as a deviation from the mean.
p: the mean product of two variables when expressed as deviations from their respective arithmetic means, i.e., $p = \frac{\Sigma(xy)}{N}$. When written with subscripts, as p_{12}, the latter relate to the variables in question, as x_1, x_2.
p': the mean product of two variables when expressed as deviations from assumed arithmetic means, i.e., $p' = \frac{\Sigma(x'y')}{N}$.
r: the Pearsonian coefficient of correlation. When written with subscripts, the latter indicate the variables to which the co-

efficient relates. Thus r_{yx} refers to the variables y and x, and r_{12} refers to the variables x_1 and x_2.

ρ(Rho): a general index of correlation (not to be confused with Spearman's coefficient of correlation based upon the squares of differences in rank). Subscripts should be employed to indicate the variables to which the measure relates, as ρ_{yx}, ρ_{xy}, $\rho_{\log yx}$, $\rho_{\log y\ \log x}$, $\rho_{1\ \bar{y}^x}$, etc. (In each case the first subscript relates to the dependent variable.)

d: the deviation of a given observation from a fitted curve; the difference between an observed and a corresponding computed value of a variable.

v: a residual; identical in meaning with d, as given above.

S: the root-mean-square deviation about a fitted curve; the standard error of estimate. This measure should be written with a subscript to indicate the variable to which it applies, as S_y, S_x, $S_{\log y}$ (the standard error of estimate in terms of logarithms), S_r (the standard error of estimate in terms of ratios), $S_{1\ \bar{y}}$ (the standard error of estimate in terms of reciprocals).

η: (Eta): the correlation ratio. Subscripts should be employed to represent the variables to which the measure relates, as η_{yx} or η_{xy}. The first subscript in each case relates to the dependent variable.

σ_{ay}: the root-mean-square deviation about a line through the means of the columns of a correlation table; the standard deviation of the y-arrays about their respective means. The symbol σ_{ax} has the same meaning with respect to the rows of a correlation table, or the x-arrays.

σ_{my}: the standard deviation of the means of the columns of a correlation table about the mean of all the y's, the mean of each column being weighted by the number of items in that column. The symbol σ_{mx} has the same meaning with respect to the means of the rows.

ζ (Zeta): the test for linearity of regression ($\zeta = \eta^2 - r^2$).

κ: the number of arrays employed in the computation of a given correlation ratio.

b: the coefficient of regression; the slope of a line of regression. When written with subscripts, the latter relate to the variables in question, as b_{yx}, b_{12} (for the variables x_1, x_2). The first subscript relates to the dependent variable in each case; b_{yx} is the coefficient of regression of y on x and b_{xy} is the coefficient of regression of x on y.

$R_{1.234}$: the coefficient of multiple correlation between a dependent variable, x_1, and a combination of independent variables, x_2,

x_3, and x_4. The order may be changed, but the primary subscript always relates to the dependent variable.

$r_{12.34}$: the coefficient of partial or net correlation between the variables x_1 and x_2, when the variables x_3 and x_4 are held constant. The order of subscripts is changed for a different combination of variables, the two primary subscripts always relating to the variables between which the net correlation is being measured.

$b_{12.34}$: the coefficient of net regression between the variables x_1 and x_2, the former being dependent, when the variables x_3 and x_4 are also taken account of in the estimating equation; the weight given to x_2 in estimating x_1, when the estimate is also based upon values of x_3 and x_4. The order of subscripts is changed for a different combination of variables.

$S_{1.234}$: the root-mean-square deviation about a line describing the relationship between a dependent variable, x_1, and a series of independent variables, x_2, x_3 and x_4; the standard error of estimate of x_1 under these conditions.

$\sigma_{1.234}$: the standard deviation of the fourth order; identical with $S_{1.234}$.

(In all the measures immediately above, the number of subscripts corresponds to the number of variables included in a given study. For the sake of simplicity, only four variables have been assumed.)

5. Symbols employed in the measurement of errors.

σ_M: the standard error of the arithmetic mean.

σ_σ: the standard error of the standard deviation. Similarly, the symbol σ, with any subscript, represents the standard error of the measure to which the subscript relates.

$P.E._M$: the probable error of the arithmetic mean ($P.E._M = .67449\ \sigma_M$). Similarly, the symbol $P.E.$ with any subscript represents the probable error of the measure to which the subscript relates.

6. Other symbols.

p: the probability of the successful outcome of a given event.

q: the probability of an unsuccessful outcome of a given event.

s: the sum of the known quantities in an observation equation; used in checking the formation and solution of the normal equations.

Δ (Delta): a general symbol for the difference between two magnitudes or between successive values of a variable quantity. The variable to which it refers may be represented by a subscript, as Δ_y. First differences are represented by Δ^1, second differences by Δ^2, etc.

ν_1, ν_2, ν_3, etc.: moments of a frequency distribution about an arbitrary origin.

π_1, π_2, π_3, etc.: uncorrected moments of a frequency distribution about the arithmetic mean.

μ_1, μ_2, μ_3, etc.: moments of a frequency distribution about the arithmetic mean after the application of Sheppard's corrections. (If Sheppard's corrections are not necessary, these symbols would be employed for the uncorrected moments about the mean.)

β_1: $\dfrac{\mu_3^2}{\mu_2^3}$

β_2: $\dfrac{\mu_4}{\mu_2^2}$

κ_2: A criterion of curve type based on β_1 and β_2.

χ^2: A quantity used in testing goodness of fit.

Greek Alphabet

Letters	Names	Letters	Names	Letters	Names
A α	Alpha	I ι	Iota	P ρ	Rho
B β	Beta	K κ	Kappa	Σ σ ς	Sigma
Γ γ	Gamma	Λ λ	Lambda	T τ	Tau
Δ δ	Delta	M μ	Mu	Υ υ	Upsilon
E ϵ	Epsilon	N ν	Nu	Φ ϕ	Phi
Z ζ	Zeta	Ξ ξ	Xi	X χ	Chi
H η	Eta	O o	Omicron	Ψ ψ	Psi
Θ θ	Theta	Π π	Pi	Ω ω	Omega

VII

MATHEMATICAL CONSTANTS AND SUMMARY OF CERTAIN MATHEMATICAL OPERATIONS

A. Mathematical Constants

$$e = 2.71828\ 18285$$

$$\frac{1}{e} = 0.36787\ 94417$$

$$\log_{10} e = 0.43429\ 44819$$

$$\pi = 3.14159\ 26536$$

$$\log_{10} \pi = 0.49714\ 98727$$

$$\sqrt{\pi} = 1.77245\ 38509$$

$$\log_{10} \sqrt{\pi} = 0.24857\ 49364$$

$$\frac{1}{\pi} = 0.31830\ 98862$$

$$\pi^2 = 9.86960\ 44011$$

$$\frac{1}{\sqrt{2\pi}} = 0.39894\ 22803$$

$$\log \frac{1}{\sqrt{2\pi}} = 9.60091\ 00657 - 10$$

$$\log_{10} \log_{10} e = 9.63778\ 43113 - 10$$

B. Summary of Mathematical Operations Involving the Employment of Exponents and Logarithms

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$(abc)^m = a^m b^m c^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^m \div a^n = a^{m-n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$a^o = 1$$

$$\log (pqr) = \log p + \log q + \log r$$

$$\log \frac{q}{p} = \log q - \log p$$

$$\log (q)^r = r \log q$$

$$\log \sqrt[q]{m} = \frac{\log m}{q}$$

VIII

TABLE A

Common Logarithms of the Natural Numbers from 1 to 11,000

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	—Infinity.	30	1.47 712	60	1.77 815	90	1.95 424	120	2.07 918
1	0.00 000	31	1.49 136	61	1.78 533	91	1.95 904	121	2.08 279
2	0.30 103	32	1.50 515	62	1.79 239	92	1.96 379	122	2.08 636
3	0.47 712	33	1.51 851	63	1.79 934	93	1.96 848	123	2.08 991
4	0.60 206	34	1.53 148	64	1.80 618	94	1.97 313	124	2.09 342
5	0.69 897	35	1.54 407	65	1.81 291	95	1.97 772	125	2.09 691
6	0.77 815	36	1.55 630	66	1.81 954	96	1.98 227	126	2.10 037
7	0.84 510	37	1.56 820	67	1.82 607	97	1.98 677	127	2.10 380
8	0.90 309	38	1.57 978	68	1.83 251	98	1.99 123	128	2.10 721
9	0.95 424	39	1.59 106	69	1.83 885	99	1.99 564	129	2.11 059
10	1.00 000	40	1.60 206	70	1.84 510	100	2.00 000	130	2.11 394
11	1.04 139	41	1.61 278	71	1.85 126	101	2.00 432	131	2.11 727
12	1.07 918	42	1.62 325	72	1.85 733	102	2.00 860	132	2.12 057
13	1.11 394	43	1.63 347	73	1.86 332	103	2.01 284	133	2.12 385
14	1.14 613	44	1.64 345	74	1.86 923	104	2.01 703	134	2.12 710
15	1.17 609	45	1.65 321	75	1.87 506	105	2.02 119	135	2.13 033
16	1.20 412	46	1.66 276	76	1.88 081	106	2.02 531	136	2.13 354
17	1.23 045	47	1.67 210	77	1.88 649	107	2.02 938	137	2.13 672
18	1.25 527	48	1.68 124	78	1.89 209	108	2.03 342	138	2.13 988
19	1.27 875	49	1.69 020	79	1.89 763	109	2.03 743	139	2.14 301
20	1.30 103	50	1.69 897	80	1.90 309	110	2.04 139	140	2.14 613
21	1.32 222	51	1.70 757	81	1.90 849	111	2.04 532	141	2.14 922
22	1.34 242	52	1.71 600	82	1.91 381	112	2.04 922	142	2.15 229
23	1.36 173	53	1.72 428	83	1.91 908	113	2.05 308	143	2.15 534
24	1.38 021	54	1.73 239	84	1.92 428	114	2.05 690	144	2.15 836
25	1.39 794	55	1.74 036	85	1.92 942	115	2.06 070	145	2.16 137
26	1.41 497	56	1.74 819	86	1.93 450	116	2.06 446	146	2.16 435
27	1.43 136	57	1.75 587	87	1.93 952	117	2.06 819	147	2.16 732
28	1.44 716	58	1 76 343	88	1.94 448	118	2.07 188	148	2.17 026
29	1.46 240	59	1.77 085	89	1.94 939	119	2.07 555	149	2.17 319
30	1.47 712	60	1.77 815	90	1.95 424	120	2.07 918	150	2.17 609

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
01	432	475	518	561	604	647	689	732	775	817
02	860	903	945	988	*030	*072	*115	*157	*199	*242
03	01 284	326	368	410	452	494	536	578	620	662
04	703	745	787	828	870	912	953	995	*036	*078
05	02 119	160	202	243	284	325	366	407	449	490
06	531	572	612	653	694	735	776	816	857	898
07	938	979	*019	*060	*100	*141	*181	*222	*262	*302
08	03 342	383	423	463	503	543	583	623	663	703
09	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
11	532	571	610	650	689	727	766	805	844	883
12	922	961	999	*038	*077	*115	*154	*192	*231	*269
13	05 308	346	385	423	461	500	538	576	614	652
14	690	729	767	805	843	881	918	956	994	*032
15	06 070	108	145	183	221	258	296	333	371	408
16	446	483	521	558	595	633	670	707	744	781
17	819	856	893	930	967	*004	*041	*078	*115	*151
18	07 188	225	262	298	335	372	408	445	482	518
19	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
21	08 279	314	350	386	422	458	493	529	565	600
22	636	672	707	743	778	814	849	884	920	955
23	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
24	09 342	377	412	447	482	517	552	587	621	656
25	691	726	760	795	830	864	899	934	968	*003
26	10 037	072	106	140	175	209	243	278	312	346
27	380	415	449	483	517	551	585	619	653	687
28	721	755	789	823	857	890	924	958	992	*025
29	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
31	727	760	793	826	860	893	926	959	992	*024
32	12 057	090	123	156	189	222	254	287	320	352
33	385	418	450	483	516	548	581	613	646	678
34	710	743	775	808	840	872	905	937	969	*001
35	13 033	066	098	130	162	194	226	258	290	322
36	354	386	418	450	481	513	545	577	609	640
37	672	704	735	767	799	830	862	893	925	956
38	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
39	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
41	922	953	983	*014	*045	*076	*106	*137	*168	*198
42	15 229	259	290	320	351	381	412	442	473	503
43	534	564	594	625	655	685	715	746	776	806
44	836	866	897	927	957	987	*017	*047	*077	*107
45	16 137	167	197	227	256	286	316	346	376	406
46	435	465	495	524	554	584	613	643	673	702
47	732	761	791	820	850	879	909	938	967	997
48	17 026	056	085	114	143	173	202	231	260	289
49	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.			
	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

N.	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
51	898	926	955	984	*013	*041	*070	*099	*127	*156
52	18 184	213	241	270	298	327	355	384	412	441
53	469	498	526	554	583	611	639	667	696	724
54	752	780	808	837	865	893	921	949	977	*005
55	19 033	061	089	117	145	173	201	229	257	285
56	312	340	368	396	424	451	479	507	535	562
57	590	618	645	673	700	728	756	783	811	838
58	866	893	921	948	976	*003	*030	*058	*085	*112
59	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
61	683	710	737	763	790	817	844	871	898	925
62	952	978	*005	*032	*059	*085	*112	*139	*165	*192
63	21 219	245	272	299	325	352	378	405	431	458
64	484	511	537	564	590	617	643	669	696	722
65	748	775	801	827	854	880	906	932	958	985
66	22 011	037	063	089	115	141	167	194	220	246
67	272	298	324	350	376	401	427	453	479	505
68	531	557	583	608	634	660	686	712	737	763
69	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
71	300	325	350	376	401	426	452	477	502	528
72	553	578	603	629	654	679	704	729	754	779
73	805	830	855	880	905	930	955	980	*005	*030
74	24 055	080	105	130	155	180	204	229	254	279
75	304	329	353	378	403	428	452	477	502	527
76	551	576	601	625	650	674	699	724	748	773
77	797	822	846	871	895	920	944	969	993	*018
78	25 042	066	091	115	139	164	188	212	237	261
79	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
81	768	792	816	840	864	888	912	935	959	983
82	26 007	031	055	079	102	126	150	174	198	221
83	245	269	293	316	340	364	387	411	435	458
84	482	505	529	553	576	600	623	647	670	694
85	717	741	764	788	811	834	858	881	905	928
86	951	975	998	*021	*045	*068	*091	*114	*138	*161
87	27 184	207	231	254	277	300	323	346	370	393
88	416	439	462	485	508	531	554	577	600	623
89	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
91	28 103	126	149	171	194	217	240	262	285	307
92	330	353	375	398	421	443	466	488	511	533
93	556	578	601	623	646	668	691	713	735	758
94	780	803	825	847	870	892	914	937	959	981
95	29 003	026	048	070	092	115	137	159	181	203
96	226	248	270	292	314	336	358	380	403	425
97	447	469	491	513	535	557	579	601	623	645
98	667	688	710	732	754	776	798	820	842	863
99	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.		
	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21 6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
01	320	341	363	384	406	428	449	471	492	514
02	535	557	578	600	621	643	664	685	707	728
03	750	771	792	814	835	856	878	899	920	942
04	963	984	*006	*027	*048	*069	*091	*112	*133	*154
05	31 175	197	218	239	260	281	302	323	345	366
06	387	408	429	450	471	492	513	534	555	576
07	597	618	639	660	681	702	723	744	765	785
08	806	827	848	869	890	911	931	952	973	994
09	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
11	428	449	469	490	510	531	552	572	593	613
12	634	654	675	695	715	736	756	777	797	818
13	838	858	879	899	919	940	960	980	*001	*021
14	33 041	062	082	102	122	143	163	183	203	224
15	244	264	284	304	325	345	365	385	405	425
16	445	465	486	506	526	546	566	586	606	626
17	646	666	686	706	726	746	766	786	806	826
18	846	866	885	905	925	945	965	985	*005	*025
19	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
21	439	459	479	498	518	537	557	577	596	616
22	635	655	674	694	713	733	753	772	792	811
23	830	850	869	889	908	928	947	967	986	*005
24	35 025	044	064	083	102	122	141	160	180	199
25	218	238	257	276	295	315	334	353	372	392
26	411	430	449	468	488	507	526	545	564	583
27	603	622	641	660	679	698	717	736	755	774
28	793	813	832	851	870	889	908	927	946	965
29	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
31	361	380	399	418	436	455	474	493	511	530
32	549	568	586	605	624	642	661	680	698	717
33	736	754	773	791	810	829	847	866	884	903
34	922	940	959	977	996	*014	*033	*051	*070	*088
35	37 107	125	144	162	181	199	218	236	254	273
36	291	310	328	346	365	383	401	420	438	457
37	475	493	511	530	548	566	585	603	621	639
38	658	676	694	712	731	749	767	785	803	822
39	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
41	202	220	238	256	274	292	310	328	346	364
42	382	399	417	435	453	471	489	507	525	543
43	561	578	596	614	632	650	668	686	703	721
44	739	757	775	792	810	828	846	863	881	899
45	917	934	952	970	987	*005	*023	*041	*058	*076
46	39 094	111	129	146	164	182	199	217	235	252
47	270	287	305	322	340	358	375	393	410	428
48	445	463	480	498	515	533	550	568	585	602
49	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.		
	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
250	39 794	811	829	846	863	881	898	915	933	950	
51	967	985	*002	*019	*037	*054	*071	*088	*106	*123	18
52	40 140	157	175	192	209	226	243	261	278	295	1 1.8
53	312	329	346	364	381	398	415	432	449	466	2 3.6
54	483	500	518	535	552	569	586	603	620	637	3 5.4
55	654	671	688	705	722	739	756	773	790	807	4 7.2
56	824	841	858	875	892	909	926	943	960	976	5 9.0
57	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	6 10.8
58	41 162	179	196	212	229	246	263	280	296	313	7 12.6
59	330	347	363	380	397	414	430	447	464	481	8 14.4
260	497	514	531	547	564	581	597	614	631	647	9 16.2
61	664	681	697	714	731	747	764	780	797	814	17
62	830	847	863	880	896	913	929	946	963	979	1 1.7
63	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	2 3.4
64	42 160	177	193	210	226	243	259	275	292	308	3 5.1
65	325	341	357	374	390	406	423	439	455	472	4 6.8
66	488	504	521	537	553	570	586	602	619	635	5 8.5
67	651	667	684	700	716	732	749	765	781	797	6 10.2
68	813	830	846	862	878	894	911	927	943	959	7 11.9
69	975	991	*008	*024	*040	*056	*072	*088	*104	*120	8 13.6
270	43 136	152	169	185	201	217	233	249	265	281	9 15.3
71	297	313	329	345	361	377	393	409	425	441	16
72	457	473	489	505	521	537	553	569	584	600	1 1.6
73	616	632	648	664	680	696	712	727	743	759	2 3.2
74	775	791	807	823	838	854	870	886	902	917	3 4.8
75	933	949	965	981	996	*012	*028	*044	*059	*075	4 6.4
76	44 091	107	122	138	154	170	185	201	217	232	5 8.0
77	248	264	279	295	311	326	342	358	373	389	6 9.6
78	404	420	436	451	467	483	498	514	529	545	7 11.2
79	560	576	592	607	623	638	654	669	685	700	8 12.8
280	716	731	747	762	778	793	809	824	840	855	9 14.4
81	871	886	902	917	932	948	963	979	994	*010	15
82	45 025	040	056	071	086	102	117	133	148	163	1 1.5
83	179	194	209	225	240	255	271	286	301	317	2 3.0
84	332	347	362	378	393	408	423	439	454	469	3 4.5
85	484	500	515	530	545	561	576	591	606	621	4 6.0
86	637	652	667	682	697	712	728	743	758	773	5 7.5
87	788	803	818	834	849	864	879	894	909	924	6 9.0
88	939	954	969	984	*000	*015	*030	*045	*060	*075	7 10.5
89	46 090	105	120	135	150	165	180	195	210	225	8 12.0
290	240	255	270	285	300	315	330	345	359	374	9 13.5
91	389	404	419	434	449	464	479	494	509	523	14
92	538	553	568	583	598	613	627	642	657	672	1 1.4
93	687	702	716	731	746	761	776	790	805	820	2 2.8
94	835	850	864	879	894	909	923	938	953	967	3 4.2
95	982	997	*012	*026	*041	*056	*070	*085	*100	*114	4 5.6
96	47 129	144	159	173	188	202	217	232	246	261	5 7.0
97	276	290	305	319	334	349	363	378	392	407	6 8.4
98	422	436	451	465	480	494	509	524	538	553	7 9.8
99	567	582	596	611	625	640	654	669	683	698	8 11.2
300	712	727	741	756	770	784	799	813	828	842	9 12.6
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
300	47 712	727	741	756	770	784	799	813	828	842	
01	857	871	885	900	914	929	943	958	972	986	
02	48 001	015	029	044	058	073	087	101	116	130	
03	144	159	173	187	202	216	230	244	259	273	15
04	287	302	316	330	344	359	373	387	401	416	1 1.5
05	430	444	458	473	487	501	515	530	544	558	2 3.0
06	572	586	601	615	629	643	657	671	686	700	3 4.5
07	714	728	742	756	770	785	799	813	827	841	4 6.0
08	855	869	883	897	911	926	940	954	968	982	5 7.5
09	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	6 9.0
310	49 136	150	164	178	192	206	220	234	248	262	7 10.5
11	276	290	304	318	332	346	360	374	388	402	8 12.0
12	415	429	443	457	471	485	499	513	527	541	9 13.5
13	554	568	582	596	610	624	638	651	665	679	
14	693	707	721	734	748	762	776	790	803	817	
15	831	845	859	872	886	900	914	927	941	955	14
16	969	982	996	*010	*024	*037	*051	*065	*079	*092	1 1.4
17	50 106	120	133	147	161	174	188	202	215	229	2 2.8
18	243	256	270	284	297	311	325	338	352	365	3 4.2
19	379	393	406	420	433	447	461	474	488	501	4 5.6
320	515	529	542	556	569	583	596	610	623	637	5 7.0
21	651	664	678	691	705	718	732	745	759	772	6 8.4
22	786	799	813	826	840	853	866	880	893	907	7 9.8
23	920	934	947	961	974	987	*001	*014	*028	*041	8 11.2
24	51 055	068	081	095	108	121	135	148	162	175	9 12.6
25	188	202	215	228	242	255	268	282	295	308	
26	322	335	348	362	375	388	402	415	428	441	
27	455	468	481	495	508	521	534	548	561	574	13
28	587	601	614	627	640	654	667	680	693	706	1 1.3
29	720	733	746	759	772	786	799	812	825	838	2 2.6
330	851	865	878	891	904	917	930	943	957	970	3 3.9
31	983	996	*009	*022	*035	*048	*061	*075	*088	*101	4 5.2
32	52 114	127	140	153	166	179	192	205	218	231	5 6.5
33	244	257	270	284	297	310	323	336	349	362	6 7.8
34	375	388	401	414	427	440	453	466	479	492	7 9.1
35	504	517	530	543	556	569	582	595	608	621	8 10.4
36	634	647	660	673	686	699	711	724	737	750	9 11.7
37	763	776	789	802	815	827	840	853	866	879	
38	892	905	917	930	943	956	969	982	994	*007	
39	53 020	033	046	058	071	084	097	110	122	135	12
340	148	161	173	186	199	212	224	237	250	263	1 1.2
41	275	288	301	314	326	339	352	364	377	390	2 2.4
42	403	415	428	441	453	466	479	491	504	517	3 3.6
43	529	542	555	567	580	593	605	618	631	643	4 4.8
44	656	668	681	694	706	719	732	744	757	769	5 6.0
45	782	794	807	820	832	845	857	870	882	895	6 7.2
46	908	920	933	945	958	970	983	995	*008	*020	7 8.4
47	54 033	045	058	070	083	095	108	120	133	145	8 9.6
48	158	170	183	195	208	220	233	245	258	270	9 10.8
49	283	295	307	320	332	345	357	370	382	394	
350	407	419	432	444	456	469	481	494	506	518	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
350	54 407	419	432	444	456	469	481	494	506	518	
51	531	543	555	568	580	593	605	617	630	642	
52	654	667	679	691	704	716	728	741	753	765	
53	777	790	802	814	827	839	851	864	876	888	13
54	900	913	925	937	949	962	974	986	998	*011	1 1.3
55	55 023	035	047	060	072	084	096	108	121	133	2 2.6
56	145	157	169	182	194	206	218	230	242	255	3 3.9
57	267	279	291	303	315	328	340	352	364	376	4 5.2
58	388	400	413	425	437	449	461	473	485	497	5 6.5
59	509	522	534	546	558	570	582	594	606	618	6 7.8
360	630	642	654	666	678	691	703	715	727	739	7 9.1
61	751	763	775	787	799	811	823	835	847	859	8 10.4
62	871	883	895	907	919	931	943	955	967	979	9 11.7
63	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
64	56 110	122	134	146	158	170	182	194	205	217	
65	229	241	253	265	277	289	301	312	324	336	12
66	348	360	372	384	396	407	419	431	443	455	1 1.2
67	467	478	490	502	514	526	538	549	561	573	2 2.4
68	585	597	608	620	632	644	656	667	679	691	3 3.6
69	703	714	726	738	750	761	773	785	797	808	4 4.8
370	820	832	844	855	867	879	891	902	914	926	5 6.0
71	937	949	961	972	984	996	*008	*019	*031	*043	6 7.2
72	57 054	066	078	089	101	113	124	136	148	159	7 8.4
73	171	183	194	206	217	229	241	252	264	276	8 9.6
74	287	299	310	322	334	345	357	368	380	392	9 10.8
75	403	415	426	438	449	461	473	484	496	507	
76	519	530	542	553	565	576	588	600	611	623	
77	634	646	657	669	680	692	703	715	726	738	11
78	749	761	772	784	795	807	818	830	841	852	1 1.1
79	864	875	887	898	910	921	933	944	955	967	2 2.2
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	3 3.3
81	58 092	104	115	127	138	149	161	172	184	195	4 4.4
82	206	218	229	240	252	263	274	286	297	309	5 5.5
83	320	331	343	354	365	377	388	399	410	422	6 6.6
84	433	444	456	467	478	490	501	512	524	535	7 7.7
85	546	557	569	580	591	602	614	625	636	647	8 8.8
86	659	670	681	692	704	715	726	737	749	760	9 9.9
87	771	782	794	805	816	827	838	850	861	872	
88	883	894	906	917	928	939	950	961	973	984	
89	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	10
390	59 106	118	129	140	151	162	173	184	195	207	1 1.0
91	218	229	240	251	262	273	284	295	306	318	2 2.0
92	329	340	351	362	373	384	395	406	417	428	3 3.0
93	439	450	461	472	483	494	506	517	528	539	4 4.0
94	550	561	572	583	594	605	616	627	638	649	5 5.0
95	660	671	682	693	704	715	726	737	748	759	6 6.0
96	770	780	791	802	813	824	835	846	857	868	7 7.0
97	879	890	901	912	923	934	945	956	966	977	8 8.0
98	988	999	*010	*021	*032	*043	*054	*065	*076	*086	9 9.0
99	60 097	108	119	130	141	152	163	173	184	195	
400	206	217	228	239	249	260	271	282	293	304	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
400	60 206	217	228	239	249	260	271	282	293	304	
01	314	325	336	347	358	369	379	390	401	412	
02	423	433	444	455	466	477	487	498	509	520	
03	531	541	552	563	574	584	595	606	617	627	
04	638	649	660	670	681	692	703	713	724	735	
05	746	756	767	778	788	799	810	821	831	842	
06	853	863	874	885	895	906	917	927	938	949	11
07	959	970	981	991	*002	*013	*023	*034	*045	*055	1 1.1
08	61 066	077	087	098	109	119	130	140	151	162	2 2.2
09	172	183	194	204	215	225	236	247	257	268	3 3.3
410	278	289	300	310	321	331	342	352	363	374	4 4.4
11	384	395	405	416	426	437	448	458	469	479	5 5.5
12	490	500	511	521	532	542	553	563	574	584	6 6.6
13	595	606	616	627	637	648	658	669	679	690	7 7.7
14	700	711	721	731	742	752	763	773	784	794	8 8.8
15	805	815	826	836	847	857	868	878	888	899	9 9.9
16	909	920	930	941	951	962	972	982	993	*003	
17	62 014	024	034	045	055	066	076	086	097	107	
18	118	128	138	149	159	170	180	190	201	211	
19	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
21	428	439	449	459	469	480	490	500	511	521	10
22	531	542	552	562	572	583	593	603	613	624	1 1.0
23	634	644	655	665	675	685	696	706	716	726	2 2.0
24	737	747	757	767	778	788	798	808	818	829	3 3.0
25	839	849	859	870	880	890	900	910	921	931	4 4.0
26	941	951	961	972	982	992	*002	*012	*022	*033	5 5.0
27	63 043	053	063	073	083	094	104	114	124	134	6 6.0
28	144	155	165	175	185	195	205	215	225	236	7 7.0
29	246	256	266	276	286	296	306	317	327	337	8 8.0
430	347	357	367	377	387	397	407	417	428	438	9 9.0
31	448	458	468	478	488	498	508	518	528	538	
32	548	558	568	579	589	599	609	619	629	639	
33	649	659	669	679	689	699	709	719	729	739	
34	749	759	769	779	789	799	809	819	829	839	
35	849	859	869	879	889	899	909	919	929	939	
36	949	959	969	979	988	998	*008	*018	*028	*038	9
37	64 048	058	068	078	088	098	108	118	128	137	1 0.9
38	147	157	167	177	187	197	207	217	227	237	2 1.8
39	246	256	266	276	286	296	306	316	326	335	3 2.7
440	345	355	365	375	385	395	404	414	424	434	4 3.6
41	444	454	464	473	483	493	503	513	523	532	5 4.5
42	542	552	562	572	582	591	601	611	621	631	6 5.4
43	640	650	660	670	680	689	699	709	719	729	7 6.3
44	738	748	758	768	777	787	797	807	816	826	8 7.2
45	836	846	856	865	875	885	895	904	914	924	9 8.1
46	933	943	953	963	972	982	992	*002	*011	*021	
47	65 031	040	050	060	070	079	089	099	108	118	
48	128	137	147	157	167	176	186	196	205	215	
49	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
450	65 321	331	341	350	360	369	379	389	398	408	
51	418	427	437	447	456	466	475	485	495	504	
52	514	523	533	543	552	562	571	581	591	600	
53	610	619	629	639	648	658	667	677	686	696	
54	706	715	725	734	744	753	763	772	782	792	
55	801	811	820	830	839	849	858	868	877	887	
56	896	906	916	925	935	944	954	963	973	982	10
57	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	1 1.0
58	66 087	096	106	115	124	134	143	153	162	172	2 2.0
59	181	191	200	210	219	229	238	247	257	266	3 3.0
460	276	285	295	304	314	323	332	342	351	361	4 4.0
61	370	380	389	398	408	417	427	436	445	455	5 5.0
62	464	474	483	492	502	511	521	530	539	549	6 6.0
63	558	567	577	586	596	605	614	624	633	642	7 7.0
64	652	661	671	680	689	699	708	717	727	736	8 8.0
65	745	755	764	773	783	792	801	811	820	829	9 9.0
66	839	848	857	867	876	885	894	904	913	922	
67	932	941	950	960	969	978	987	997	*006	*015	
68	67 025	034	043	052	062	071	080	089	099	108	
69	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	9
71	302	311	321	330	339	348	357	367	376	385	1 0.9
72	394	403	413	422	431	440	449	459	468	477	2 1.8
73	486	495	504	514	523	532	541	550	560	569	3 2.7
74	578	587	596	605	614	624	633	642	651	660	4 3.6
75	669	679	688	697	706	715	724	733	742	752	5 4.5
76	761	770	779	788	797	806	815	825	834	843	6 5.4
77	852	861	870	879	888	897	906	916	925	934	7 6.3
78	943	952	961	970	979	988	997	*006	*015	*024	8 7.2
79	68 034	043	052	061	070	079	088	097	106	115	9 8.1
480	124	133	142	151	160	169	178	187	196	205	
81	215	224	233	242	251	260	269	278	287	296	
82	305	314	323	332	341	350	359	368	377	386	
83	395	404	413	422	431	440	449	458	467	476	
84	485	494	502	511	520	529	538	547	556	565	
85	574	583	592	601	610	619	628	637	646	655	
86	664	673	681	690	699	708	717	726	735	744	8
87	753	762	771	780	789	797	806	815	824	833	1 0.8
88	842	851	860	869	878	886	895	904	913	922	2 1.6
89	931	940	949	958	966	975	984	993	*002	*011	3 2.4
490	69 020	028	037	046	055	064	073	082	090	099	4 3.2
91	108	117	126	135	144	152	161	170	179	188	5 4.0
92	197	205	214	223	232	241	249	258	267	276	6 4.8
93	285	294	302	311	320	329	338	346	355	364	7 5.6
94	373	381	390	399	408	417	425	434	443	452	8 6.4
95	461	469	478	487	496	504	513	522	531	539	9 7.2
96	548	557	566	574	583	592	601	609	618	627	
97	636	644	653	662	671	679	688	697	705	714	
98	723	732	740	749	758	767	775	784	793	801	
99	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
500	69 897	906	914	923	932	940	949	958	966	975	
01	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
02	70 070	079	088	096	105	114	122	131	140	148	
03	157	165	174	183	191	200	209	217	226	234	
04	243	252	260	269	278	286	295	303	312	321	
05	329	338	346	355	364	372	381	389	398	406	
06	415	424	432	441	449	458	467	475	484	492	9
07	501	509	518	526	535	544	552	561	569	578	1 0.9
08	586	595	603	612	621	629	638	646	655	663	2 1.8
09	672	680	689	697	706	714	723	731	740	749	3 2.7
510	757	766	774	783	791	800	808	817	825	834	4 3.6
11	842	851	859	868	876	885	893	902	910	919	5 4.5
12	927	935	944	952	961	969	978	986	995	*003	6 5.4
13	71 012	020	029	037	046	054	063	071	079	088	7 6.3
14	096	105	113	122	130	139	147	155	164	172	8 7.2
15	181	189	198	206	214	223	231	240	248	257	9 8.1
16	265	273	282	290	299	307	315	324	332	341	
17	349	357	366	374	383	391	399	408	416	425	
18	433	441	450	458	466	475	483	492	500	508	
19	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
21	684	692	700	709	717	725	734	742	750	759	8
22	767	775	784	792	800	809	817	825	834	842	1 0.8
23	850	858	867	875	883	892	900	908	917	925	2 1.6
24	933	941	950	958	966	975	983	991	999	*008	3 2.4
25	72 016	024	032	041	049	057	066	074	082	090	4 3.2
26	099	107	115	123	132	140	148	156	165	173	5 4.0
27	181	189	198	206	214	222	230	239	247	255	6 4.8
28	263	272	280	288	296	304	313	321	329	337	7 5.6
29	346	354	362	370	378	387	395	403	411	419	8 6.4
530	428	436	444	452	460	469	477	485	493	501	9 7.2
31	509	518	526	534	542	550	558	567	575	583	
32	591	599	607	616	624	632	640	648	656	665	
33	673	681	689	697	705	713	722	730	738	746	
34	754	762	770	779	787	795	803	811	819	827	
35	835	843	852	860	868	876	884	892	900	908	
36	916	925	933	941	949	957	965	973	981	989	7
37	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	1 0.7
38	73 078	086	094	102	111	119	127	135	143	151	2 1.4
39	159	167	175	183	191	199	207	215	223	231	3 2.1
540	239	247	255	263	272	280	288	296	304	312	4 2.8
41	320	328	336	344	352	360	368	376	384	392	5 3.5
42	400	408	416	424	432	440	448	456	464	472	6 4.2
43	480	488	496	504	512	520	528	536	544	552	7 4.9
44	560	568	576	584	592	600	608	616	624	632	8 5.6
45	640	648	656	664	672	679	687	695	703	711	9 6.3
46	719	727	735	743	751	759	767	775	783	791	
47	799	807	815	823	830	838	846	854	862	870	
48	878	886	894	902	910	918	926	933	941	949	
49	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
550	74 036	044	052	060	068	076	084	092	099	107	
51	115	123	131	139	147	155	162	170	178	186	
52	194	202	210	218	225	233	241	249	257	265	
53	273	280	288	296	304	312	320	327	335	343	
54	351	359	367	374	382	390	398	406	414	421	
55	429	437	445	453	461	468	476	484	492	500	
56	507	515	523	531	539	547	554	562	570	578	
57	586	593	601	609	617	624	632	640	648	656	
58	663	671	679	687	695	702	710	718	726	733	
59	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
61	896	904	912	920	927	935	943	950	958	966	8
62	974	981	989	997	*003	*012	*020	*028	*035	*043	1 0.8
63	75 051	059	066	074	082	089	097	105	113	120	2 1.6
64	128	136	143	151	159	166	174	182	189	197	3 2.4
65	205	213	220	228	236	243	251	259	266	274	4 3.2
66	282	289	297	305	312	320	328	335	343	351	5 4.0
67	358	366	374	381	389	397	404	412	420	427	6 4.8
68	435	442	450	458	465	473	481	488	496	504	7 5.6
69	511	519	526	534	542	549	557	565	572	580	8 6.4
570	587	595	603	610	618	626	633	641	648	656	9 7.2
71	664	671	679	686	694	702	709	717	724	732	
72	740	747	755	762	770	778	785	793	800	808	
73	815	823	831	838	846	853	861	868	876	884	
74	891	899	906	914	921	929	937	944	952	959	
75	967	974	982	989	997	*005	*012	*020	*027	*035	
76	76 042	050	057	065	072	080	087	095	103	110	
77	118	125	133	140	148	155	163	170	178	185	
78	193	200	208	215	223	230	238	245	253	260	
79	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
81	418	425	433	440	448	455	462	470	477	485	7
82	492	500	507	515	522	530	537	545	552	559	1 0.7
83	567	574	582	589	597	604	612	619	626	634	2 1.4
84	641	649	656	664	671	678	686	693	701	708	3 2.1
85	716	723	730	738	745	753	760	768	775	782	4 2.8
86	790	797	805	812	819	827	834	842	849	856	5 3.5
87	864	871	879	886	893	901	908	916	923	930	6 4.2
88	938	945	953	960	967	975	982	989	997	*004	7 4.9
89	77 012	019	026	034	041	048	056	063	070	078	8 5.6
590	085	093	100	107	115	122	129	137	144	151	9 6.3
91	159	166	173	181	188	195	203	210	217	225	
92	232	240	247	254	262	269	276	283	291	298	
93	305	313	320	327	335	342	349	357	364	371	
94	379	386	393	401	408	415	422	430	437	444	
95	452	459	466	474	481	488	495	503	510	517	
96	525	532	539	546	554	561	568	576	583	590	
97	597	605	612	619	627	634	641	648	656	663	
98	670	677	685	692	699	706	714	721	728	735	
99	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
600	77 815	822	830	837	844	851	859	866	873	880	
01	887	895	902	909	916	924	931	938	945	952	
02	960	967	974	981	988	996	*003	*010	*017	*025	
03	78 032	039	046	053	061	068	075	082	089	097	
04	104	111	118	125	132	140	147	154	161	168	
05	176	183	190	197	204	211	219	226	233	240	
06	247	254	262	269	276	283	290	297	305	312	8
07	319	326	333	340	347	353	362	369	376	383	1 0.8
08	390	398	405	412	419	426	433	440	447	455	2 1.6
09	462	469	476	483	490	497	504	512	519	526	3 2.4
610	533	540	547	554	561	569	576	583	590	597	4 3.2
11	604	611	618	625	633	640	647	654	661	668	5 4.0
12	675	682	689	696	704	711	718	725	732	739	6 4.8
13	746	753	760	767	774	781	789	796	803	810	7 5.6
14	817	824	831	838	845	852	859	866	873	880	8 6.4
15	888	895	902	909	916	923	930	937	944	951	9 7.2
16	958	965	972	979	986	993	*000	*007	*014	*021	
17	79 029	036	043	050	057	064	071	078	085	092	
18	099	106	113	120	127	134	141	148	155	162	
19	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
21	309	316	323	330	337	344	351	358	365	372	7
22	379	386	393	400	407	414	421	428	435	442	1 0.7
23	449	456	463	470	477	484	491	498	505	511	2 1.4
24	518	525	532	539	546	553	560	567	574	581	3 2.1
25	588	595	602	609	616	623	630	637	644	650	4 2.8
26	657	664	671	678	685	692	699	706	713	720	5 3.5
27	727	734	741	748	754	761	768	775	782	789	6 4.2
28	796	803	810	817	824	831	837	844	851	858	7 4.9
29	865	872	879	886	893	900	906	913	920	927	8 5.6
630	934	941	948	955	962	969	975	982	989	996	9 6.3
31	80 003	010	017	024	030	037	044	051	058	065	
32	072	079	085	092	099	106	113	120	127	134	
33	140	147	154	161	168	175	182	188	195	202	
34	209	216	223	229	236	243	250	257	264	271	
35	277	284	291	298	305	312	318	325	332	339	
36	346	353	359	366	373	380	387	393	400	407	6
37	414	421	428	434	441	448	455	462	468	475	1 0.6
38	482	489	496	502	509	516	523	530	536	543	2 1.2
39	550	557	564	570	577	584	591	598	604	611	3 1.8
640	618	625	632	638	645	652	659	665	672	679	4 2.4
41	686	693	699	706	713	720	726	733	740	747	5 3.0
42	754	760	767	774	781	787	794	801	808	814	6 3.6
43	821	828	835	841	848	855	862	868	875	882	7 4.2
44	889	895	902	909	916	922	929	936	943	949	8 4.8
45	956	963	969	976	983	990	996	*003	*010	*017	9 5.4
46	81 023	030	037	043	050	057	064	070	077	084	
47	090	097	104	111	117	124	131	137	144	151	
48	158	164	171	178	184	191	198	204	211	218	
49	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
N.	0	1	2	3	4	5	6	7	8	9	Prop Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
650	81 291	298	305	311	318	325	331	338	345	351	
51	358	365	371	378	385	391	398	405	411	418	
52	425	431	438	445	451	458	465	471	478	485	
53	491	498	505	511	518	525	531	538	544	551	
54	558	564	571	578	584	591	598	604	611	617	
55	624	631	637	644	651	657	664	671	677	684	
56	690	697	704	710	717	723	730	737	743	750	
57	757	763	770	776	783	790	796	803	809	816	
58	823	829	836	842	849	856	862	869	875	882	
59	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
61	82 020	027	033	040	046	053	060	066	073	079	7
62	086	092	099	105	112	119	125	132	138	145	1 0.7
63	151	158	164	171	178	184	191	197	204	210	2 1.4
64	217	223	230	236	243	249	256	263	269	276	3 2.1
65	282	289	295	302	308	315	321	328	334	341	4 2.8
66	347	354	360	367	373	380	387	393	400	406	5 3.5
67	413	419	426	432	439	445	452	458	465	471	6 4.2
68	478	484	491	497	504	510	517	523	530	536	7 4.9
69	543	549	556	562	569	575	582	588	595	601	8 5.6
670	607	614	620	627	633	640	646	653	659	666	9 6.3
71	672	679	685	692	698	705	711	718	724	730	
72	737	743	750	756	763	769	776	782	789	795	
73	802	808	814	821	827	834	840	847	853	860	
74	866	872	879	885	892	898	905	911	918	924	
75	930	937	943	950	956	963	969	975	982	988	
76	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
77	83 059	065	072	078	085	091	097	104	110	117	
78	123	129	136	142	149	155	161	168	174	181	
79	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
81	315	321	327	334	340	347	353	359	366	372	6
82	378	385	391	398	404	410	417	423	429	436	1 0.6
83	442	448	455	461	467	474	480	487	493	499	2 1.2
84	506	512	518	525	531	537	544	550	556	563	3 1.8
85	569	575	582	588	594	601	607	613	620	626	4 2.4
86	632	639	645	651	658	664	670	677	683	689	5 3.0
87	696	702	708	715	721	727	734	740	746	753	6 3.6
88	759	765	771	778	784	790	797	803	809	816	7 4.2
89	822	828	835	841	847	853	860	866	872	879	8 4.8
690	885	891	897	904	910	916	923	929	935	942	9 5.4
91	948	954	960	967	973	979	985	992	998	*004	
92	84 011	017	023	029	036	042	048	055	061	067	
93	073	080	086	092	098	105	111	117	123	130	
94	136	142	148	155	161	167	173	180	186	192	
95	198	205	211	217	223	230	236	242	248	255	
96	261	267	273	280	286	292	298	305	311	317	
97	323	330	336	342	348	354	361	367	373	379	
98	386	392	398	404	410	417	423	429	435	442	
99	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
700	84 510	516	522	528	535	541	547	553	559	566	
01	572	578	584	590	597	603	609	615	621	628	
02	634	640	646	652	658	665	671	677	683	689	
03	696	702	708	714	720	726	733	739	745	751	
04	757	763	770	776	782	788	794	800	807	813	
05	819	825	831	837	844	850	856	862	868	874	
06	880	887	893	899	905	911	917	924	930	936	7
07	942	948	954	960	967	973	979	985	991	997	1 0.7
08	85 003	009	016	022	028	034	040	046	052	058	2 1.4
09	065	071	077	083	089	095	101	107	114	120	3 2.1
710	126	132	138	144	150	156	163	169	175	181	4 2.8
11	187	193	199	205	211	217	224	230	236	242	5 3.5
12	248	254	260	266	272	278	285	291	297	303	6 4.2
13	309	315	321	327	333	339	345	352	358	364	7 4.9
14	370	376	382	388	394	400	406	412	418	425	8 5.6
15	431	437	443	449	455	461	467	473	479	485	9 6.3
16	491	497	503	509	516	522	528	534	540	546	
17	552	558	564	570	576	582	588	594	600	606	
18	612	618	625	631	637	643	649	655	661	667	
19	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
21	794	800	806	812	818	824	830	836	842	848	6
22	854	860	866	872	878	884	890	896	902	908	1 0.6
23	914	920	926	932	938	944	950	956	962	968	2 1.2
24	974	980	986	992	998	*004	*010	*016	*022	*028	3 1.8
25	86 034	040	046	052	058	064	070	076	082	088	4 2.4
26	094	100	106	112	118	124	130	136	141	147	5 3.0
27	153	159	165	171	177	183	189	195	201	207	6 3.6
28	213	219	225	231	237	243	249	255	261	267	7 4.2
29	273	279	285	291	297	303	308	314	320	326	8 4.8
730	332	338	344	350	356	362	368	374	380	386	9 5.4
31	392	398	404	410	415	421	427	433	439	445	
32	451	457	463	469	475	481	487	493	499	504	
33	510	516	522	528	534	540	546	552	558	564	
34	570	576	581	587	593	599	605	611	617	623	
35	629	635	641	646	652	658	664	670	676	682	
36	688	694	700	705	711	717	723	729	735	741	5
37	747	753	759	764	770	776	782	788	794	800	1 0.5
38	806	812	817	823	829	835	841	847	853	859	2 1.0
39	864	870	876	882	888	894	900	906	911	917	3 1.5
740	923	929	935	941	947	953	958	964	970	976	4 2.0
41	982	988	994	999	*005	*011	*017	*023	*029	*035	5 2.5
42	87 040	046	052	058	064	070	075	081	087	093	6 3.0
43	099	105	111	116	122	128	134	140	146	151	7 3.5
44	157	163	169	175	181	186	192	198	204	210	8 4.0
45	216	221	227	233	239	245	251	256	262	268	9 4.6
46	274	280	286	291	297	303	309	315	320	326	
47	332	338	344	349	355	361	367	373	379	384	
48	390	396	402	408	413	419	425	431	437	442	
49	448	454	460	466	471	477	483	489	495	500	
750	506	512	518	523	529	535	541	547	552	558	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
750	87 506	512	518	523	529	535	541	547	552	558	
51	564	570	576	581	587	593	599	604	610	616	
52	622	628	633	639	645	651	656	662	668	674	
53	679	685	691	697	703	708	714	720	726	731	
54	737	743	749	754	760	766	772	777	783	789	
55	795	800	806	812	818	823	829	835	841	846	
56	852	858	864	869	875	881	887	892	898	904	
57	910	915	921	927	933	938	944	950	955	961	
58	967	973	978	984	990	996	*001	*007	*013	*018	
59	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
61	138	144	150	156	161	167	173	178	184	190	6
62	195	201	207	213	218	224	230	235	241	247	1 0.6
63	252	258	264	270	275	281	287	292	298	304	2 1.2
64	309	315	321	326	332	338	343	349	355	360	3 1.8
65	366	372	377	383	389	395	400	406	412	417	4 2.4
66	423	429	434	440	446	451	457	463	468	474	5 3.0
67	480	485	491	497	502	508	513	519	525	530	6 3.6
68	536	542	547	553	559	564	570	576	581	587	7 4.2
69	593	598	604	610	615	621	627	632	638	643	8 4.8
770	649	655	660	666	672	677	683	689	694	700	9 5.4
71	705	711	717	722	728	734	739	745	750	756	
72	762	767	773	779	784	790	795	801	807	812	
73	818	824	829	835	840	846	852	857	863	868	
74	874	880	885	891	897	902	908	913	919	925	
75	930	936	941	947	953	958	964	969	975	981	
76	986	992	997	*003	*009	*014	*020	*025	*031	*037	
77	89 042	048	053	059	064	070	076	081	087	092	
78	098	104	109	115	120	126	131	137	143	148	
79	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
81	265	271	276	282	287	293	298	304	310	315	5
82	321	326	332	337	343	348	354	360	365	371	1 0.5
83	376	382	387	393	398	404	409	415	421	426	2 1.0
84	432	437	443	448	454	459	465	470	476	481	3 1.5
85	487	492	498	504	509	515	520	526	531	537	4 2.0
86	542	548	553	559	564	570	575	581	586	592	5 2.5
87	597	603	609	614	620	625	631	636	642	647	6 3.0
88	653	658	664	669	675	680	686	691	697	702	7 3.5
89	708	713	719	724	730	735	741	746	752	757	8 4.0
790	763	768	774	779	785	790	796	801	807	812	9 4.5
91	818	823	829	834	840	845	851	856	862	867	
92	873	878	883	889	894	900	905	911	916	922	
93	927	933	938	944	949	955	960	966	971	977	
94	982	988	993	998	*004	*009	*015	*020	*026	*031	
95	90 037	042	048	053	059	064	069	075	080	086	
96	091	097	102	108	113	119	124	129	135	140	
97	146	151	157	162	168	173	179	184	189	195	
98	200	206	211	217	222	227	233	238	244	249	
99	255	260	266	271	276	282	287	293	298	304	
800	309	314	320	325	331	336	342	347	352	358	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
800	90 309	314	320	325	331	336	342	347	352	358	
01	363	369	374	380	385	390	396	401	407	412	
02	417	423	428	434	439	445	450	455	461	466	
03	472	477	482	488	493	499	504	509	515	520	
04	526	531	536	542	547	553	558	563	569	574	
05	580	585	590	596	601	607	612	617	623	628	
06	634	639	644	650	655	660	666	671	677	682	
07	687	693	698	703	709	714	720	725	730	736	
08	741	747	752	757	763	768	773	779	784	789	
09	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	865	870	875	881	886	891	897	
11	902	907	913	918	924	929	934	940	945	950	6
12	956	961	966	972	977	982	988	993	998	*004	1 0.6
13	91 009	014	020	025	030	036	041	046	052	057	2 1.2
14	062	068	073	078	084	089	094	100	105	110	3 1.8
15	116	121	126	132	137	142	148	153	158	164	4 2.4
16	169	174	180	185	190	196	201	206	212	217	5 3.0
17	222	228	233	238	243	249	254	259	265	270	6 3.6
18	275	281	286	291	297	302	307	312	318	323	7 4.2
19	328	334	339	344	350	355	360	365	371	376	8 4.8
820	381	387	392	397	403	408	413	418	424	429	9 5.4
21	434	440	445	450	455	461	466	471	477	482	
22	487	492	498	503	508	514	519	524	529	535	
23	540	545	551	556	561	566	572	577	582	587	
24	593	598	603	609	614	619	624	630	635	640	
25	645	651	656	661	666	672	677	682	687	693	
26	698	703	709	714	719	724	730	735	740	745	
27	751	756	761	766	772	777	782	787	793	798	
28	803	808	814	819	824	829	834	840	845	850	
29	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
31	960	965	971	976	981	986	991	997	*002	*007	5
32	92 012	018	023	028	033	038	044	049	054	059	1 0.5
33	065	070	075	080	085	091	096	101	106	111	2 1.0
34	117	122	127	132	137	143	148	153	158	163	3 1.5
35	169	174	179	184	189	195	200	205	210	215	4 2.0
36	221	226	231	236	241	247	252	257	262	267	5 2.5
37	273	278	283	288	293	298	304	309	314	319	6 3.0
38	324	330	335	340	345	350	355	361	366	371	7 3.5
39	376	381	387	392	397	402	407	412	418	423	8 4.0
840	428	433	438	443	449	454	459	464	469	474	9 4.5
41	480	485	490	495	500	505	511	516	521	526	
42	531	536	542	547	552	557	562	567	572	578	
43	583	588	593	598	603	609	614	619	624	629	
44	634	639	645	650	655	660	665	670	675	681	
45	686	691	696	701	706	711	716	722	727	732	
46	737	742	747	752	758	763	768	773	778	783	
47	788	793	799	804	809	814	819	824	829	834	
48	840	845	850	855	860	865	870	875	881	886	
49	891	896	901	906	911	916	921	927	932	937	
850	942	947	952	957	962	967	973	978	983	988	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
850	92 942	947	952	957	962	967	973	978	983	988	
51	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
52	93 044	049	054	059	064	069	075	080	085	090	
53	095	100	105	110	115	120	125	131	136	141	
54	146	151	156	161	166	171	176	181	186	192	
55	197	202	207	212	217	222	227	232	237	242	
56	247	252	258	263	268	273	278	283	288	293	6
57	298	303	308	313	318	323	328	334	339	344	1 0.6
58	349	354	359	364	369	374	379	384	389	394	2 1.2
59	399	404	409	414	420	425	430	435	440	445	3 1.8
860	450	455	460	465	470	475	480	485	490	495	4 2.4
61	500	505	510	515	520	526	531	536	541	546	5 3.0
62	551	556	561	566	571	576	581	586	591	596	6 3.6
63	601	606	611	616	621	626	631	636	641	646	7 4.2
64	651	656	661	666	671	676	682	687	692	697	8 4.8
65	702	707	712	717	722	727	732	737	742	747	9 5.4
66	752	757	762	767	772	777	782	787	792	797	
67	802	807	812	817	822	827	832	837	842	847	
68	852	857	862	867	872	877	882	887	892	897	
69	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
71	94 002	007	012	017	022	027	032	037	042	047	5
72	052	057	062	067	072	077	082	086	091	096	1 0.5
73	101	106	111	116	121	126	131	136	141	146	2 1.0
74	151	156	161	166	171	176	181	186	191	196	3 1.5
75	201	206	211	216	221	226	231	236	240	245	4 2.0
76	250	255	260	265	270	275	280	285	290	295	5 2.5
77	300	305	310	315	320	325	330	335	340	345	6 3.0
78	349	354	359	364	369	374	379	384	389	394	7 3.5
79	399	404	409	414	419	424	429	433	438	443	8 4.0
880	448	453	458	463	468	473	478	483	488	493	9 4.5
81	498	503	507	512	517	522	527	532	537	542	
82	547	552	557	562	567	571	576	581	586	591	
83	596	601	606	611	616	621	626	630	635	640	
84	645	650	655	660	665	670	675	680	685	689	
85	694	699	704	709	714	719	724	729	734	738	
86	743	748	753	758	763	768	773	778	783	787	4
87	792	797	802	807	812	817	822	827	832	836	1 0.4
88	841	846	851	856	861	866	871	876	880	885	2 0.8
89	890	895	900	905	910	915	919	924	929	934	3 1.2
890	939	944	949	954	959	963	968	973	978	983	4 1.6
91	988	993	998	*002	*007	*012	*017	*022	*027	*032	5 2.0
92	95 036	041	046	051	056	061	066	071	075	080	6 2.4
93	085	090	095	100	105	109	114	119	124	129	7 2.8
94	134	139	143	148	153	158	163	168	173	177	8 3.2
95	182	187	192	197	202	207	211	216	221	226	9 3.6
96	231	236	240	245	250	255	260	265	270	274	
97	279	284	289	294	299	303	308	313	318	323	
98	328	332	337	342	347	352	357	361	366	371	
99	376	381	386	390	395	400	405	410	415	419	
900	424	429	434	439	444	448	453	458	463	468	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
900	95 424	429	434	439	444	448	453	458	463	468	
01	472	477	482	487	492	497	501	506	511	516	
02	521	525	530	535	540	545	550	554	559	564	
03	569	574	578	583	588	593	598	602	607	612	
04	617	622	626	631	636	641	646	650	655	660	
05	665	670	674	679	684	689	694	698	703	708	
06	713	718	722	727	732	737	742	746	751	756	
07	761	766	770	775	780	785	789	794	799	804	
08	809	813	818	823	828	832	837	842	847	852	
09	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
11	952	957	961	966	971	976	980	985	990	995	5
12	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	1 0.5
13	96 047	052	057	061	066	071	076	080	085	090	2 1.0
14	095	099	104	109	114	118	123	128	133	137	3 1.5
15	142	147	152	156	161	166	171	175	180	185	4 2.0
16	190	194	199	204	209	213	218	223	227	232	5 2.5
17	237	242	246	251	256	261	265	270	275	280	6 3.0
18	284	289	294	298	303	308	313	317	322	327	7 3.5
19	332	336	341	346	350	355	360	365	369	374	8 4.0
920	379	384	388	393	398	402	407	412	417	421	9 4.5
21	426	431	435	440	445	450	454	459	464	468	
22	473	478	483	487	492	497	501	506	511	515	
23	520	525	530	534	539	544	548	553	558	562	
24	567	572	577	581	586	591	595	600	605	609	
25	614	619	624	628	633	638	642	647	652	656	
26	661	666	670	675	680	685	689	694	699	703	
27	708	713	717	722	727	731	736	741	745	750	
28	755	759	764	769	774	778	783	788	792	797	
29	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
31	895	900	904	909	914	918	923	928	932	937	4
32	942	946	951	956	960	965	970	974	979	984	1 0.4
33	988	993	997	*002	*007	*011	*016	*021	*025	*030	2 0.8
34	97 035	039	044	049	053	058	063	067	072	077	3 1.2
35	081	086	090	095	100	104	109	114	118	123	4 1.6
36	128	132	137	142	146	151	155	160	165	169	5 2.0
37	174	179	183	188	192	197	202	206	211	216	6 2.4
38	220	225	230	234	239	243	248	253	257	262	7 2.8
39	267	271	276	280	285	290	294	299	304	308	8 3.2
940	313	317	322	327	331	336	340	345	350	354	9 3.6
41	359	364	368	373	377	382	387	391	396	400	
42	405	410	414	419	424	428	433	437	442	447	
43	451	456	460	465	470	474	479	483	488	493	
44	497	502	506	511	516	520	525	529	534	539	
45	543	548	552	557	562	566	571	575	580	585	
46	589	594	598	603	607	612	617	621	626	630	
47	635	640	644	649	653	658	663	667	672	676	
48	681	685	690	695	699	704	708	713	717	722	
49	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
950	97 772	777	782	786	791	795	800	804	809	813	
51	818	823	827	832	836	841	845	850	855	859	
52	864	868	873	877	882	886	891	896	900	905	
53	909	914	918	923	928	932	937	941	946	950	
54	953	959	964	968	973	978	982	987	991	996	
55	98 000	005	009	014	019	023	028	032	037	041	
56	046	050	055	059	064	068	073	078	082	087	
57	091	096	100	105	109	114	118	123	127	132	
58	137	141	146	150	155	159	164	168	173	177	
59	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
61	272	277	281	286	290	295	299	304	308	313	5
62	318	322	327	331	336	340	345	349	354	358	1 0.5
63	363	367	372	376	381	385	390	394	399	403	2 1.0
64	408	412	417	421	426	430	435	439	444	448	3 1.5
65	453	457	462	466	471	475	480	484	489	493	4 2.0
66	498	502	507	511	516	520	525	529	534	538	5 2.5
67	543	547	552	556	561	565	570	574	579	583	6 3.0
68	588	592	597	601	605	610	614	619	623	628	7 3.5
69	632	637	641	646	650	655	659	664	668	673	8 4.0
970	677	682	686	691	695	700	704	709	713	717	9 4.5
71	722	726	731	735	740	744	749	753	758	762	
72	767	771	776	780	784	789	793	798	802	807	
73	811	816	820	825	829	834	838	843	847	851	
74	856	860	865	869	874	878	883	887	892	896	
75	900	905	909	914	918	923	927	932	936	941	
76	945	949	954	958	963	967	972	976	981	985	
77	989	994	998	*003	*007	*012	*016	*021	*025	*029	
78	99 034	038	043	047	052	056	061	065	069	074	
79	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
81	167	171	176	180	185	189	193	198	202	207	4
82	211	216	220	224	229	233	238	242	247	251	1 0.4
83	255	260	264	269	273	277	282	286	291	295	2 0.8
84	300	304	308	313	317	322	326	330	335	339	3 1.2
85	344	348	352	357	361	366	370	374	379	383	4 1.6
86	388	392	396	401	405	410	414	419	423	427	5 2.0
87	432	436	441	445	449	454	458	463	467	471	6 2.4
88	476	480	484	489	493	498	502	506	511	515	7 2.8
89	520	524	528	533	537	542	546	550	555	559	8 3.2
990	564	568	572	577	581	585	590	594	599	603	9 3.6
91	607	612	616	621	625	629	634	638	642	647	
92	651	656	660	664	669	673	677	682	686	691	
93	695	699	704	708	712	717	721	726	730	734	
94	739	743	747	752	756	760	765	769	774	778	
95	782	787	791	795	800	804	808	813	817	822	
96	826	830	835	839	843	848	852	856	861	865	
97	870	874	878	883	887	891	896	900	904	909	
98	913	917	922	926	930	935	939	944	948	952	
99	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
1000	000 000	043	087	130	174	217	260	304	347	391	
1001	434	477	521	564	608	651	694	738	781	824	
1002	868	911	954	998	*041	*084	*128	*171	*214	*258	
1003	001 301	344	388	431	474	517	561	604	647	690	**44**
1004	734	777	820	863	907	950	993	*036	*080	*123	1 4.4
1005	002 166	209	252	296	339	382	425	468	512	555	2 8.8
1006	598	641	684	727	771	814	857	900	943	986	3 13.2
1007	003 029	073	116	159	202	245	288	331	374	417	4 17.6
1008	461	504	547	590	633	676	719	762	805	848	5 22.0
1009	891	934	977	*020	*063	*106	*149	*192	*235	*278	6 26.4
1010	004 321	364	407	450	493	536	579	622	665	708	7 30.8
1011	751	794	837	880	923	966	*009	*052	*095	*138	8 35.2
1012	005 180	223	266	309	352	395	438	481	524	567	9 39.6
1013	609	652	695	738	781	824	867	909	952	995	
1014	006 038	081	124	166	209	252	295	338	380	423	
1015	466	509	552	594	637	680	723	765	808	851	**43**
1016	894	936	979	*022	*065	*107	*150	*193	*236	*278	1 4.3
1017	007 321	364	406	449	492	534	577	620	662	705	2 8.6
1018	748	790	833	876	918	961	*004	*046	*089	*132	3 12.9
1019	008 174	217	259	302	345	387	430	472	515	558	4 17.2
1020	600	643	685	728	770	813	856	898	941	983	5 21.5
1021	009 026	068	111	153	196	238	281	323	366	408	6 25.8
1022	451	493	536	578	621	663	706	748	791	833	7 30.1
1023	876	918	961	*003	*045	*088	*130	*173	*215	*258	8 34.4
1024	010 300	342	385	427	470	512	554	597	639	681	9 38.7
1025	724	766	809	851	893	936	978	*020	*063	*105	
1026	**011** 147	190	232	274	317	359	401	444	486	528	
1027	570	613	655	697	740	782	824	866	909	951	**42**
1028	993	*035	*078	*120	*162	*204	*247	*289	*331	*373	1 4.2
1029	012 415	458	500	542	584	626	669	711	753	795	2 8.4
1030	837	879	922	964	*006	*048	*090	*132	*174	*217	3 12.6
1031	013 259	301	343	385	427	469	511	553	596	638	4 16.8
1032	680	722	764	806	848	890	932	974	*016	*058	5 21.0
1033	014 100	142	184	226	268	310	352	395	437	479	6 25.2
1034	521	563	605	647	689	730	772	814	856	898	7 29.4
1035	940	982	*024	*066	*108	*150	*192	*234	*276	*318	8 33.6
1036	015 360	402	444	485	527	569	611	653	695	737	9 37.8
1037	779	821	863	904	946	988	*030	*072	*114	*156	
1038	016 197	239	281	323	365	407	448	490	532	574	
1039	616	657	699	741	783	824	866	908	950	992	**41**
1040	017 033	075	117	159	200	242	284	326	367	409	1 4.1
1041	451	492	534	576	618	659	701	743	784	826	2 8.2
1042	868	909	951	993	*034	*076	*118	*159	*201	*243	3 12.3
1043	018 284	326	368	409	451	492	534	576	617	659	4 16.4
1044	700	742	784	825	867	908	950	992	*033	*075	5 20.5
1045	019 116	158	199	241	282	324	366	407	449	490	6 24.6
1046	532	573	615	656	698	739	781	822	864	905	7 28.7
1047	947	988	*030	*071	*113	*154	*195	*237	*278	*320	8 32.8
1048	020 361	403	444	486	527	568	610	651	693	734	**9 36.9**
1049	775	817	858	900	941	982	*024	*065	*107	*148	
1050	021 189	231	272	313	355	396	437	479	520	561	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
1050	021 189	231	272	313	355	396	437	479	520	561	
1051	603	644	685	727	768	809	851	892	933	974	
1052	022 016	057	098	140	181	222	263	305	346	387	
1053	428	470	511	552	593	635	676	717	758	799	42
1054	841	882	923	964	*005	*047	*088	*129	*170	*211	1 4.2
1055	023 252	294	335	376	417	458	499	541	582	623	2 8.4
1056	664	705	746	787	828	870	911	952	993	*034	3 12.6
1057	024 075	116	157	198	239	280	321	363	404	445	4 16.8
1058	486	527	568	609	650	691	732	773	814	855	5 21.0
1059	896	937	978	*019	*060	*101	*142	*183	*224	*265	6 25.2
1060	025 306	347	388	429	470	511	552	593	634	674	7 29.4
1061	715	756	797	838	879	920	961	*002	*043	*084	8 33.6
1062	026 125	165	206	247	288	329	370	411	452	492	9 37.8
1063	533	574	615	656	697	737	778	819	860	901	
1064	942	982	*023	*064	*105	*146	*186	*227	*268	*309	
1065	027 350	390	431	472	513	553	594	635	676	716	41
1066	757	798	839	879	920	961	*002	*042	*083	*124	1 4.1
1067	028 164	205	246	287	327	368	409	449	490	531	2 8.2
1068	571	612	653	693	734	775	815	856	896	937	3 12.3
1069	978	*018	*059	*100	*140	*181	*221	*262	*303	*343	4 16.4
1070	029 384	424	465	506	546	587	627	668	708	749	5 20.5
1071	789	830	871	911	952	992	*033	*073	*114	*154	6 24.6
1072	030 195	235	276	316	357	397	438	478	519	559	7 28.7
1073	600	640	681	721	762	802	843	883	923	964	8 32.8
1074	031 004	045	085	126	166	206	247	287	328	368	9 36.9
1075	408	449	489	530	570	610	651	691	732	772	
1076	812	853	893	933	974	*014	*054	*095	*135	*175	
1077	032 216	256	296	337	377	417	458	498	538	578	40
1078	619	659	699	740	780	820	860	901	941	981	1 4.0
1079	033 021	062	102	142	182	223	263	303	343	384	2 8.0
1080	424	464	504	544	585	625	665	705	745	785	3 12.0
1081	826	866	906	946	986	*027	*067	*107	*147	*187	4 16.0
1082	034 227	267	308	348	388	428	468	508	548	588	5 20.0
1083	628	669	709	749	789	829	869	909	949	989	6 24.0
1084	035 029	069	109	149	190	230	270	310	350	390	7 28.0
1085	430	470	510	550	590	630	670	710	750	790	8 32.0
1086	830	870	910	950	990	*030	*070	*110	*150	*190	9 36.0
1087	036 230	269	309	349	389	429	469	509	549	589	
1088	629	669	709	749	789	828	868	908	948	988	
1089	037 028	068	108	148	187	227	267	307	347	387	39
1090	426	466	506	546	586	626	665	705	745	785	1 3.9
1091	825	865	904	944	984	*024	*064	*103	*143	*183	2 7.8
1092	038 223	262	302	342	382	421	461	501	541	580	3 11.7
1093	620	660	700	739	779	819	859	898	938	978	4 15.6
1094	039 017	057	097	136	176	216	255	295	335	374	5 19.5
1095	414	454	493	533	573	612	652	692	731	771	6 23.4
1096	811	850	890	929	969	*009	*048	*088	*127	*167	7 27.3
1097	040 207	246	286	325	365	405	444	484	523	563	8 31.2
1098	602	642	681	721	761	800	840	879	919	958	9 35.1
1099	998	*037	*077	*116	*156	*195	*235	*274	*314	*353	
1100	041 393	432	472	511	551	590	630	669	708	748	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

TABLE B

Squares, Square Roots and Reciprocals of the Natural Numbers from 1 to 1000

n	n^2	$n^{1/2}$	$1/n$
1	1	1.000 0000	1.000 000 000
2	4	1.414 2136	0.500 000 000
3	9	1.732 0508	.333 333 333
4	16	2.000 0000	.250 000 000
5	25	2.236 0680	.200 000 000
6	36	2.449 4897	.166 666 667
7	49	2.645 7513	.142 857 143
8	64	2.828 4271	.125 000 000
9	81	3.000 0000	.111 111 111
10	1 00	3.162 2777	.100 000 000
11	1 21	3.316 6248	.090 909 091
12	1 44	3.464 1016	.083 333 333
13	1 69	3.605 5513	.076 923 077
14	1 96	3.741 6574	.071 428 571
15	2 25	3.872 9833	.066 666 667
16	2 56	4.000 0000	.062 500 000
17	2 89	4.123 1056	.058 823 529
18	3 24	4.242 6407	.055 555 556
19	3 61	4.358 8989	.052 631 579
20	4 00	4.472 1360	.050 000 000
21	4 41	4.582 5757	.047 619 048
22	4 84	4.690 4158	.045 454 545
23	5 29	4.795 8315	.043 478 261
24	5 76	4.898 9795	.041 666 667
25	6 25	5.000 0000	.040 000 000
26	6 76	5.099 0195	.038 461 538
27	7 29	5.196 1524	.037 037 037
28	7 84	5.291 5026	.035 714 286
29	8 41	5.385 1648	.034 482 759
30	9 00	5.477 2256	.033 333 333
31	9 61	5.567 7644	.032 258 065
32	10 24	5.656 8542	.031 250 000
33	10 89	5.744 5626	.030 303 030
34	11 56	5.830 9519	.029 411 765
35	12 25	5.916 0798	.028 571 429
36	12 96	6.000 0000	.027 777 778
37	13 69	6.082 7625	.027 027 027
38	14 44	6.164 4140	.026 315 789
39	15 21	6.244 9980	.025 641 026
40	16 00	6.324 5553	.025 000 000
41	16 81	6.403 1242	.024 390 244
42	17 64	6.480 7407	.023 809 524
43	18 49	6.557 4385	.023 255 814
44	19 36	6.633 2496	.022 727 273
45	20 25	6.708 2039	.022 222 222
46	21 16	6.782 3300	.021 739 130
47	22 09	6.855 6546	.021 276 596
48	23 04	6.928 2032	.020 833 333
49	24 01	7.000 0000	.020 408 163
50	25 00	7.071 0678	.020 000 000

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
51	26 01	7.141 4284	.019 607 843
52	27 04	7.211 1026	.019 230 769
53	28 09	7.280 1099	.018 867 925
54	29 16	7.348 4692	.018 518 519
55	30 25	7.416 1985	.018 181 818
56	31 36	7.483 3148	.017 857 143
57	32 49	7.549 8344	.017 543 860
58	33 64	7.615 7731	.017 241 379
59	34 81	7.681 1457	.016 949 153
60	36 00	7.745 9667	.016 666 667
61	37 21	7.810 2497	.016 393 443
62	38 44	7.874 0079	.016 129 032
63	39 69	7.937 2539	.015 873 016
64	40 96	8.000 0000	.015 625 000
65	42 25	8.062 2577	.015 384 615
66	43 56	8.124 0384	.015 151 515
67	44 89	8.185 3528	.014 925 373
68	46 24	8.246 2113	.014 705 882
69	47 61	8.306 6239	.014 492 754
70	49 00	8.366 6003	.014 285 714
71	50 41	8.426 1498	.014 084 507
72	51 84	8.485 2814	.013 888 889
73	53 29	8.544 0037	.013 698 630
74	54 76	8.602 3253	.013 513 514
75	56 25	8.660 2540	.013 333 333
76	57 76	8.717 7979	.013 157 895
77	59 29	8.774 9644	.012 987 013
78	60 84	8.831 7609	.012 820 513
79	62 41	8.888 1944	.012 658 228
80	64 00	8.944 2719	.012 500 000
81	65 61	9.000 0000	.012 345 679
82	67 24	9.055 3851	.012 195 122
83	68 89	9.110 4336	.012 048 193
84	70 56	9.165 1514	.011 904 762
85	72 25	9.219 5445	.011 764 706
86	73 96	9.273 6185	.011 627 907
87	75 69	9.327 3791	.011 494 253
88	77 44	9.380 8315	.011 363 636
89	79 21	9.433 9811	.011 235 955
90	81 00	9.486 8330	.011 111 111
91	82 81	9.539 3920	.010 989 011
92	84 64	9.591 6630	.010 869 565
93	86 49	9.643 6508	.010 752 688
94	88.36	9.695 3597	.010 638 298
95	90 25	9.746 7943	.010 526 316
96	92 16	9.797 9590	.010 416 667
97	94 09	9.848 8578	.010 309 278
98	96 04	9.899 4949	.010 204 082
99	98 01	9.949 8744	.010 101 010
100	1 00 00	10.000 0000	.010 000 000

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
101	1 02 01	10.049 8756	.009 900 990
102	1 04 04	10.099 5049	.009 803 922
103	1 06 09	10.148 8916	.009 708 738
104	1 08 16	10.198 0390	.009 615 385
105	1 10 25	10.246 9508	.009 523 810
106	1 12 36	10.295 6301	.009 433 962
107	1 14 49	10.344 0804	.009 345 794
108	1 16 64	10.392 3048	.009 259 259
109	1 18 81	10.440 3065	.009 174 312
110	1 21 00	10.488 0885	.009 090 909
111	1 23 21	10.535 6538	.009 009 009
112	1 25 44	10.583 0052	.008 928 571
113	1 27 69	10.630 1458	.008 849 558
114	1 29 96	10.677 0783	.008 771 930
115	1 32 25	10.723 8053	.008 695 652
116	1 34 56	10.770 3296	.008 620 690
117	1 36 89	10.816 6538	.008 547 009
118	1 39 24	10.862 7805	.008 474 576
119	1 41 61	10.908 7121	.008 403 361
120	1 44 00	10.954 4512	.008 333 333
121	1 46 41	11.000 0000	.008 264 463
122	1 48 84	11.045 3610	.008 196 721
123	1 51 29	11.090 5365	.008 130 081
124	1 53 76	11.135 5287	.008 064 516
125	1 56 25	11.180 3399	.008 000 000
126	1 58 76	11.224 9722	.007 936 508
127	1 61 29	11.269 4277	.007 874 016
128	1 63 84	11.313 7085	.007 812 500
129	1 66 41	11.357 8167	.007 751 938
130	1 69 00	11.401 7543	.007 692 308
131	1 71 61	11.445 5231	.007 633 588
132	1 74 24	11.489 1253	.007 575 758
133	1 76 89	11.532 5626	.007 518 797
134	1 79 56	11.575 8369	.007 462 687
135	1 82 25	11.618 9500	.007 407 407
136	1 84 96	11.661 9038	.007 352 941
137	1 87 69	11.704 6999	.007 299 270
138	1 90 44	11.747 3401	.007 246 377
139	1 93 21	11.789 8261	.007 194 245
140	1 96 00	11.832 1596	.007 142 857
141	1 98 81	11.874 3422	.007 092 199
142	2 01 64	11.916 3753	.007 042 254
143	2 04 49	11.958 2607	.006 993 007
144	2 07 36	12.000 0000	.006 944 444
145	2 10 25	12.041 5946	.006 896 552
146	2 13 16	12.083 0460	.006 849 315
147	2 16 09	12.124 3557	.006 802 721
148	2 19 04	12.165 5251	.006 756 757
149	2 22 01	12.206 5556	.006 711 409
150	2 25 00	12.247 4487	.006 666 667

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
151	2 28 01	12.288 2057	.006 622 517
152	2 31 04	12.328 8280	.006 578 947
153	2 34 09	12.369 3169	.006 535 948
154	2 37 16	12.409 6736	.006 493 506
155	2 40 25	12.449 8996	.006 451 613
156	2 43 36	12.489 9960	.006 410 256
157	2 46 49	12.529 9641	.006 369 427
158	2 49 64	12.569 8051	.006 329 114
159	2 52 81	12.609 5202	.006 289 308
160	2 56 00	12.649 1106	.006 250 000
161	2 59 21	12.688 5775	.006 211 180
162	2 62 44	12.727 9221	.006 172 840
163	2 65 69	12.767 1453	.006 134 969
164	2 68 96	12.806 2485	.006 097 561
165	2 72 25	12.845 2326	.006 060 606
166	2 75 56	12.884 0987	.006 024 096
167	2 78 89	12.922 8480	.005 988 024
168	2 82 24	12.961 4814	.005 952 381
169	2 85 61	13.000 0000	.005 917 160
170	2 89 00	13.038 4048	.005 882 353
171	2 92 41	13.076 6968	.005 847 953
172	2 95 84	13.114 8770	.005 813 953
173	2 99 29	13.152 9464	.005 780 347
174	3 02 76	13.190 9060	.005 747 126
175	3 06 25	13.228 7566	.005 714 286
176	3 09 76	13.266 4992	.005 681 818
177	3 13 29	13.304 1347	.005 649 718
178	3 16 84	13.341 6641	.005 617 978
179	3 20 41	13.379 0882	.005 586 592
180	3 24 00	13.416 4079	.005 555 556
181	3 27 61	13.453 6240	.005 524 862
182	3 31 24	13.490 7376	.005 494 505
183	3 34 89	13.527 7493	.005 464 481
184	3 38 56	13.564 6600	.005 434 783
185	3 42 25	13.601 4705	.005 405 405
186	3 45 96	13.638 1817	.005 376 344
187	3 49 69	13.674 7943	.005 347 594
188	3 53 44	13.711 3092	.005 319 149
189	3 57 21	13.747 7271	.005 291 005
190	3 61 00	13.784 0488	.005 263 158
191	3 64 81	13.820 2750	.005 235 602
192	3 68 64	13.856 4065	.005 208 333
193	3 72 49	13.892 4440	.005 181 347
194	3 76 36	13.928 3883	.005 154 639
195	3 80 25	13.964 2400	.005 128 205
196	3 84 16	14.000 0000	.005 102 041
197	3 88 09	14.035 6688	.005 076 142
198	3 92 04	14.071 2473	.005 050 505
199	3 96 01	14.106 7360	.005 025 126
200	4 00 00	14.142 1356	.005 000 000

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
201	4 04 01	14.177 4469	.004 975 124
202	4 08 04	14.212 6704	.004 950 495
203	4 12 09	14.247 8068	.004 926 108
204	4 16 16	14.282 8569	.004 901 961
205	4 20 25	14.317 8211	.004 878 049
206	4 24 36	14.352 7001	.004 854 369
207	4 28 49	14.387 4946	.004 830 918
208	4 32 64	14.422 2051	.004 807 692
209	4 36 81	14.456 8323	.004 784 689
210	4 41 00	14.491 3767	.004 761 905
211	4 45 21	14.525 8390	.004 739 336
212	4 49 44	14.560 2198	.004 716 981
213	4 53 69	14.594 5195	.004 694 836
214	4 57 96	14.628 7388	.004 672 897
215	4 62 25	14.662 8783	.004 651 163
216	4 66 56	14.696 9385	.004 629 630
217	4 70 89	14.730 9199	.004 608 295
218	4 75 24	14.764 8231	.004 587 156
219	4 79 61	14.798 6486	004 566 210
220	4 84 00	14.832 3970	.004 545 455
221	4 88 41	14.866 0687	.004 524 887
222	4 92 84	14.899 6644	.004 504 505
223	4 97 29	14.933 1845	.004 484 305
224	5 01 76	14.966 6295	.004 464 286
225	5 06 25	15.000 0000	.004 444 444
226	5 10 76	15.033 2964	.004 424 779
227	5 15 29	15.066 5192	.004 405 286
228	5 19 84	15.099 6689	.004 385 965
229	5 24 41	15.132 7460	.004 366 812
230	5 29 00	15.165 7509	.004 347 826
231	5 33 61	15.198 6842	.004 329 004
232	5 38 24	15.231 5462	.004 310 345
233	5 42 89	15.264 3375	.004 291 845
234	5 47 56	15.297 0585	.004 273 504
235	5 52 25	15.329 7097	.004 255 319
236	5 56 96	15.362 2915	.004 237 288
237	5 61 69	15.394 8043	.004 219 409
238	5 66 44	15.427 2486	.004 201 681
239	5 71 21	15.459 6248	.004 184 100
240	5 76 00	15.491 9334	.004 166 667
241	5 80 81	15.524 1747	.004 149 378
242	5 85 64	15.556 3492	.004 132 231
243	5 90 49	15.588 4573	.004 115 226
244	5 95 36	15.620 4994	.004 098 361
245	6 00 25	15.652 4758	.004 081 633
246	6 05 16	15.684 3871	.004 065 041
247	6 10 09	15.716 2336	.004 048 583
248	6 15 04	15.748 0157	.004 032 258
249	6 20 01	15.779 7338	.004 016 064
250	6 25 00	15.811 3883	.004 000 000

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
251	6 30 01	15.842 9795	.003 984 064
252	6 35 04	15.874 5079	.003 968 254
253	6 40 09	15.905 9737	.003 952 569
254	6 45 16	15.937 3775	.003 937 008
255	6 50 25	15.968 7194	.003 921 569
256	6 55 36	16.000 0000	.003 906 250
257	6 60 49	16.031 2195	.003 891 051
258	6 65 64	16.062 3784	.003 875 969
259	6 70 81	16.093 4769	.003 861 004
260	6 76 00	16.124 5155	.003 846 154
261	6 81 21	16.155 4944	.003 831 418
262	6 86 44	16.186 4141	.003 816 794
263	6 91 69	16.217 2747	.003 802 281
264	6 96 96	16.248 0768	.003 787 879
265	7 02 25	16.278 8206	.003 773 585
266	7 07 56	16.309 5064	.003 759 398
267	7 12 89	16.340 1346	.003 745 318
268	7 18 24	16.370 7055	.003 731 343
269	7 23 61	16.401 2195	.003 717 472
270	7 29 00	16.431 6767	.003 703 704
271	7 34 41	16.462 0776	.003 690 037
272	7 39 84	16.492 4225	.003 676 471
273	7 45 29	16.522 7116	.003 663 004
274	7 50 76	16.552 9454	.003 649 635
275	7 56 25	16.583 1240	.003 636 364
276	7 61 76	16.613 2477	.003 623 188
277	7 67 29	16.643 3170	.003 610 108
278	7 72 84	16.673 3320	.003 597 122
279	7 78 41	16.703 2931	.003 584 229
280	7 84 00	16.733 2005	.003 571 429
281	7 89 61	16.763 0546	.003 558 719
282	7 95 24	16.792 8556	.003 546 099
283	8 00 89	16.822 6038	.003 533 569
284	8 06 56	16.852 2995	.003 521 127
285	8 12 25	16.881 9430	.003 508 772
286	8 17 96	16.911 5345	.003 496 503
287	8 23 69	16.941 0743	.003 484 321
288	8 29 44	16.970 5627	.003 472 222
289	8 35 21	17.000 0000	.003 460 208
290	8 41 00	17.029 3864	.003 448 276
291	8 46 81	17.058 7221	.003 436 426
292	8 52 64	17.088 0075	.003 424 658
293	8 58 49	17.117 2428	.003 412 969
294	8 64 36	17.146 4282	.003 401 361
295	8 70 25	17.175 5640	.003 389 831
296	8 76 16	17.204 6505	.003 378 378
297	8 82 09	17.233 6879	.003 367 003
298	8 88 04	17.262 6765	.003 355 705
299	8 94 01	17.291 6165	.003 344 482
300	9 00 00	17.320 5081	.003 333 333

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
301	9 06 01	17.349 3516	.003 322 259
302	9 12 04	17.378 1472	.003 311 258
303	9 18 09	17.406 8952	.003 300 330
304	9 24 16	17.435 5958	.003 289 474
305	9 30 25	17.464 2492	.003 278 689
306	9 36 36	17.492 8557	.003 267 974
307	9 42 49	17.521 4155	.003 257 329
308	9 48 64	17.549 9288	.003 246 753
309	9 54 81	17.578 3958	.003 236 246
310	9 61 00	17.606 8169	.003 225 806
311	9 67 21	17.635 1921	.003 215 434
312	9 73 44	17.663 5217	.003 205 128
313	9 79 69	17.691 8060	.003 194 888
314	9 85 96	17.720 0451	.003 184 713
315	9 92 25	17.748 2393	.003 174 603
316	9 98 56	17.776 3888	.003 164 557
317	10 04 89	17.804 4938	.003 154 574
318	10 11 24	17.832 5545	.003 144 654
319	10 17 61	17.860 5711	.003 134 796
320	10 24 00	17.888 5438	.003 125 000
321	10 30 41	17.916 4729	.003 115 265
322	10 36 84	17.944 3584	.003 105 590
323	10 43 29	17.972 2008	.003 095 975
324	10 49 76	18.000 0000	.003 086 420
325	10 56 25	18.027 7564	.003 076 923
326	10 62 76	18.055 4701	.003 067 485
327	10 69 29	18.083 1413	.003 058 104
328	10 75 84	18.110 7703	.003 048 780
329	10 82 41	18.138 3571	.003 039 514
330	10 89 00	18.165 9021	.003 030 303
331	10 95 61	18.193 4054	.003 021 148
332	11 02 24	18.220 8672	.003 012 048
333	11 08 89	18.248 2876	.003 003 003
334	11 15 56	18.275 6669	.002 994 012
335	11 22 25	18.303 0052	.002 985 075
336	11 28 96	18.330 3028	.002 976 190
337	11 35 69	18.357 5598	.002 967 359
338	11 42 44	18.384 7763	.002 958 580
339	11 49 21	18.411 9526	.002 949 853
340	11 56 00	18.439 0889	.002 941 176
341	11 62 81	18.466 1853	.002 932 551
342	11 69 64	18.493 2420	.002 923 977
343	11 76 49	18.520 2592	.002 915 452
344	11 83 36	18.547 2370	.002 906 977
345	11 90 25	18.574 1756	.002 898 551
346	11 97 16	18.601 0752	.002 890 173
347	12 04 09	18.627 9360	.002 881 844
348	12 11 04	18.654 7581	.002 873 563
349	12 18 01	18.681 5417	.002 865 330
350	12 25 00	18.708 2869	.002 857 143

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
351	12 32 01	18.734 9940	.002 849 003
352	12 39 04	18.761 6630	.002 840 909
353	12 46 09	18.788 2942	.002 832 861
354	12 53 16	18.814 8877	.002 824 859
355	12 60 25	18.841 4437	.002 816 901
356	12 67 36	18.867 9623	.002 808 989
357	12 74 49	18.894 4436	.002 801 120
358	12 81 64	18.920 8879	.002 793 296
359	12 88 81	18.947 2953	.002 785 515
360	12 96 00	18.973 6660	.002 777 778
361	13 03 21	19.000 0000	.002 770 083
362	13 10 44	19.026 2976	.002 762 431
363	13 17 69	19.052 5589	.002 754 821
364	13 24 96	19.078 7840	.002 747 253
365	13 32 25	19.104 9732	.002 739 726
366	13 39 56	19.131 1265	.002 732 240
367	13 46 89	19.157 2441	.002 724 796
368	13 54 24	19.183 3261	.002 717 391
369	13 61 61	19.209 3727	.002 710 027
370	13 69 00	19.235 3841	.002 702 703
371	13 76 41	19.261 3603	.002 695 418
372	13 83 84	19.287 3015	.002 688 172
373	13 91 29	19.313 2079	.002 680 965
374	13 98 76	19.339 0796	.002 673 797
375	14 06 25	19.364 9167	.002 666 667
376	14 13 76	19.390 7194	.002 659 574
377	14 21 29	19.416 4878	.002 652 520
378	14 28 84	19.442 2221	.002 645 503
379	14 36 41	19.467 9223	.002 638 522
380	14 44 00	19.493 5887	.002 631 579
381	14 51 61	19.519 2213	.002 624 672
382	14 59 24	19.544 8203	.002 617 801
383	14 66 89	19.570 3858	.002 610 966
384	14 74 56	19.595 9179	.002 604 167
385	14 82 25	19.621 4169	.002 597 403
386	14 89 96	19.646 8827	.002 590 674
387	14 97 69	19.672 3156	.002 583 979
388	15 05 44	19.697 7156	.002 577 320
389	15 13 21	19.723 0829	.002 570 694
390	15 21 00	19.748 4177	.002 564 103
391	15 28 81	19.773 7199	.002 557 545
392	15 36 64	19.798 9899	.002 551 020
393	15 44 49	19.824 2276	.002 544 529
394	15 52 36	19.849 4332	.002 538 071
395	15 60 25	19.874 6069	.002 531 646
396	15 68 16	19.899 7487	.002 525 253
397	15 76 09	19.924 8588	.002 518 892
398	15 84 04	19.949 9373	.002 512 563
399	15 92 01	19.974 9844	.002 506 266
400	16 00 00	20.000 0000	.002 500 000

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
401	16 08 01	20.024 9844	.002 493 766
402	16 16 04	20.049 9377	.002 487 562
403	16 24 09	20.074 8599	.002 481 390
404	16 32 16	20.099 7512	.002 475 248
405	16 40 25	20.124 6118	.002 469 136
406	16 48 36	20.149 4417	.002 463 054
407	16 56 49	20.174 2410	.002 457 002
408	16 64 64	20.199 0099	.002 450 980
409	16 72 81	20.223 7484	.002 444 988
410	16 81 00	20.248 4567	.002 439 024
411	16 89 21	20.273 1349	.002 433 090
412	16 97 44	20.297 7831	.002 427 184
413	17 05 69	20.322 4014	.002 421 308
414	17 13 96	20.346 9899	.002 415 459
415	17 22 25	20.371 5488	.002 409 639
416	17 30 56	20.396 0781	.002 403 846
417	17 38 89	20.420 5779	.002 398 082
418	17 47 24	20.445 0483	.002 392 344
419	17 55 61	20.469 4895	.002 386 635
420	17 64 00	20.493 9015	.002 380 952
421	17 72 41	20.518 2845	.002 375 297
422	17 80 84	20.542 6386	.002 369 668
423	17 89 29	20.566 9638	.002 364 066
424	17 97 76	20.591 2603	.002 358 491
425	18 06 25	20.615 5281	.002 352 941
426	18 14 76	20.639 7674	.002 347 418
427	18 23 29	20.663 9783	.002 341 920
428	18 31 84	20.688 1609	.002 336 449
429	18 40 41	20.712 3152	.002 331 002
430	18 49 00	20.736 4414	.002 325 581
431	18 57 61	20.760 5395	.002 320 186
432	18 66 24	20.784 6097	.002 314 815
433	18 74 89	20.808 6520	.002 309 469
434	18 83 56	20.832 6667	.002 304 147
435	18 92 25	20.856 6536	.002 298 851
436	19 00 96	20.880 6130	.002 293 578
437	19 09 69	20.904 5450	.002 288 330
438	19 18 44	20.928 4495	.002 283 105
439	19 27 21	20.952 3268	.002 277 904
440	19 36 00	20.976 1770	.002 272 727
441	19 44 81	21.000 0000	.002 267 574
442	19 53 64	21.023 7960	.002 262 443
443	19 62 49	21.047 5652	.002 257 336
444	19 71 36	21.071 3075	.002 252 252
445	19 80 25	21.095 0231	.002 247 191
446	19 89 16	21.118 7121	.002 242 152
447	19 98 09	21.142 3745	.002 237 136
448	20 07 04	21.166 0105	.002 232 143
449	20 16 01	21.189 6201	.002 227 171
450	20 25 00	21.213 2034	.002 222 222

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
451	20 34 01	21.236 7606	.002 217 295
452	20 43 04	21.260 2916	.002 212 389
453	20 52 09	21.283 7967	.002 207 506
454	20 61 16	21.307 2758	.002 202 643
455	20 70 25	21.330 7290	.002 197 802
456	20 79 36	21.354 1565	.002 192 982
457	20 88 49	21.377 5583	.002 188 184
458	20 97 64	21.400 9346	.002 183 406
459	21 06 81	21.424 2853	.002 178 649
460	21 16 00	21.447 6106	.002 173 913
461	21 25 21	21.470 9106	.002 169 197
462	21 34 44	21.494 1853	.002 164 502
463	21 43 69	21.517 4348	.002 159 827
464	21 52 96	21.540 6592	.002 155 172
465	21 62 25	21.563 8587	.002 150 538
466	21 71 56	21.587 0331	.002 145 923
467	21 80 89	21.610 1828	.002 141 328
468	21 90 24	21.633 3077	.002 136 752
469	21 99 61	21.656 4078	.002 132 196
470	22 09 00	21.679 4834	.002 127 660
471	22 18 41	21.702 5344	.002 123 142
472	22 27 84	21.725 5610	.002 118 644
473	22 37 29	21.748 5632	.002 114 165
474	22 46 76	21.771 5411	.002 109 705
475	22 56 25	21.794 4947	.002 105 263
476	22 65 76	21.817 4242	.002 100 840
477	22 75 29	21.840 3297	.002 096 436
478	22 84 84	21.863 2111	.002 092 050
479	22 94 41	21.886 0686	.002 087 683
480	23 04 00	21.908 9023	.002 083 333
481	23 13 61	21.931 7122	.002 079 002
482	23 23 24	21.954 4984	.002 074 689
483	23 32 89	21.977 2610	.002 070 393
484	23 42 56	22.000 0000	.002 066 116
485	23 52 25	22.022 7155	.002 061 856
486	23 61 96	22.045 4077	.002 057 613
487	23 71 69	22.068 0765	.002 053 388
488	23 81 44	22.090 7220	.002 049 180
489	23 91 21	22.113 3444	.002 044 990
490	24 01 00	22.135 9436	.002 040 816
491	24 10 81	22.158 5198	.002 036 660
492	24 20 64	22.181 0730	.002 032 520
493	24 30 49	22.203 6033	.002 028 398
494	24 40 36	22.226 1108	.002 024 291
495	24 50 25	22.248 5955	.002 020 202
496	24 60 16	22.271 0575	.002 016 129
497	24 70 09	22.293 4968	.002 012 072
498	24 80 04	22.315 9136	.002 008 032
499	24 90 01	22.338 3079	.002 004 008
500	25 00 00	22.360 6798	.002 000 000

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
501	25 10 01	22.383 0293	.001 996 008
502	25 20 04	22.405 3565	.001 992 032
503	25 30 09	22.427 6615	.001 988 072
504	25 40 16	22.449 9443	.001 984 127
505	25 50 25	22.472 2051	.001 980 198
506	25 60 36	22.494 4438	.001 976 285
507	25 70 49	22.516 6605	.001 972 387
508	25 80 64	22.538 8553	.001 968 504
509	25 90 81	22.561 0283	.001 964 637
510	26 01 00	22.583 1796	.001 960 784
511	26 11 21	22.605 3091	.001 956 947
512	26 21 44	22.627 4170	.001 953 125
513	26 31 69	22.649 5033	.001 949 318
514	26 41 96	22.671 5681	.001 945 525
515	26 52 25	22.693 6114	.001 941 748
516	26 62 56	22.715 6334	.001 937 984
517	26 72 89	22.737 6340	.001 934 236
518	26 83 24	22.759 6134	.001 930 502
519	26 93 61	22.781 5715	.001 926 782
520	27 04 00	22.803 5085	.001 923 077
521	27 14 41	22.825 4244	.001 919 386
522	27 24 84	22.847 3193	.001 915 709
523	27 35 29	22.869 1933	.001 912 046
524	27 45 76	22.891 0463	.001 908 397
525	27 56 25	22.912 8785	.001 904 762
526	27 66 76	22.934 6899	.001 901 141
527	27 77 29	22.956 4806	.001 897 533
528	27 87 84	22.978 2506	.001 893 939
529	27 98 41	23.000 0000	.001 890 359
530	28 09 00	23.021 7289	.001 886 792
531	28 19 61	23.043 4372	.001 883 239
532	28 30 24	23.065 1252	.001 879 699
533	28 40 89	23.086 7928	.001 876 173
534	28 51 56	23.108 4400	.001 872 659
535	28 62 25	23.130 0670	.001 869 159
536	28 72 96	23.151 6738	.001 865 672
537	28 83 69	23.173 2605	.001 862 197
538	28 94 44	23.194 8270	.001 858 736
539	29 05 21	23.216 3735	.001 855 288
540	29 16 00	23.237 9001	.001 851 852
541	29 26 81	23.259 4067	.001 848 429
542	29 37 64	23.280 8935	.001 845 018
543	29 48 49	23.302 3604	.001 841 621
544	29 59 36	23.323 8076	.001 838 235
545	29 70 25	23.345 2351	.001 834 862
546	29 81 16	23.366 6429	.001 831 502
547	29 92 09	23.388 0311	.001 828 154
548	30 03 04	23.409 3998	.001 824 818
549	30 14 01	23.430 7490	.001 821 494
550	30 25 00	23.452 0788	.001 818 182

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
551	30 36 01	23.473 3892	.001 814 882
552	30 47 04	23.494 6802	.001 811 594
553	30 58 09	23.515 9520	.001 808 318
554	30 69 16	23.537 2046	.001 805 054
555	30 80 25	23.558 4380	.001 801 802
556	30 91 36	23.579 6522	.001 798 561
557	31 02 49	23.600 6474	.001 795 332
558	31 13 64	23.622 0236	.001 792 115
559	31 24 81	23.643 1808	.001 788 909
560	31 36 00	23.664 3191	.001 785 714
561	31 47 21	23.685 4386	.001 782 531
562	31 58 44	23.706 5392	.001 779 359
563	31 69 69	23.727 6210	.001 776 199
564	31 80 96	23.748 6842	.001 773 050
565	31 92 25	23.769 7286	.001 769 912
566	32 03 56	23.790 7545	.001 766 784
567	32 14 89	23.811 7618	.001 763 668
568	32 26 24	23.832 7506	.001 760 563
569	32 37 61	23.853 7209	.001 757 469
570	32 49 00	23.874 6728	.001 754 386
571	32 60 41	23.895 6063	.001 751 313
572	32 71 84	23.916 0304	.001 748 252
573	32 83 29	23.937 4184	.001 745 201
574	32 94 76	23.958 2971	.001 742 160
575	33 06 25	23.979 1576	.001 739 130
576	33 17 76	24.000 0000	.001 736 111
577	33 29 29	24.020 8243	.001 733 102
578	33 40 84	24.041 6306	.001 730 104
579	33 52 41	24.062 4188	.001 727 116
580	33 64 00	24.083 1891	.001 724 138
581	33 75 61	24.103 9416	.001 721 170
582	33 87 24	24.124 6762	.001 718 213
583	33 98 89	24.145 3929	.001 715 266
584	34 10 56	24.166 0919	.001 712 329
585	34 22 25	24.186 7732	.001 709 402
586	34 33 96	24.207 4369	.001 706 485
587	34 45 69	24.228 0829	.001 703 578
588	34 57 44	24.248 7113	.001 700 680
589	34 69 21	24.269 3222	.001 697 793
590	34 81 00	24.289 9156	.001 694 915
591	34 92 81	24.310 4916	.001 692 047
592	35 04 64	24.331 0501	.001 689 189
593	35 16 49	24.351 5913	.001 686 341
594	35 28 36	24.372 1152	.001 683 502
595	35 40 25	24.392 6218	.001 680 672
596	35 52 16	24.413 1112	.001 677 852
597	35 64 09	24.433 5834	.001 675 042
598	35 76 04	24.454 0385	.001 672 241
599	35 88 01	24.474 4765	.001 669 449
600	36 00 00	24.494 8974	.001 666 667

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
601	36 12 01	24.515 3013	.001 663 894
602	36 24 04	24.535 6883	.001 661 130
603	36 36 09	24.556 0583	.001 658 375
604	36 48 16	24.576 4115	.001 655 629
605	36 60 25	24.596 7478	.001 652 893
606	36 72 36	24.617 0673	.001 650 165
607	36 84 49	24.637 3700	.001 647 446
608	36 96 64	24.657 6560	.001 644 737
609	37 08 81	24.677 9254	.001 642 036
610	37 21 00	24.698 1781	.001 639 344
611	37 33 21	24.718 4142	.001 636 661
612	37 45 44	24.738 6338	.001 633 987
613	37 57 69	24.758 8368	.001 631 321
614	37 69 96	24.779 0234	.001 628 664
615	37 82 25	24.799 1935	.001 626 016
616	37 94 56	24.819 3473	.001 623 377
617	38 06 89	24.839 4847	.001 620 746
618	38 19 24	24.859 6058	.001 618 123
619	38 31 61	24.879 7106	.001 615 509
620	38 44 00	24.899 7992	.001 612 903
621	38 56 41	24.919 8716	.001 610 306
622	38 68 84	24.939 9278	.001 607 717
623	38 81 29	24.959 9679	.001 605 136
624	38 93 76	24.979 9920	.001 602 564
625	39 06 25	25.000 0000	.001 600 000
626	39 18 76	25.019 9920	.001 597 444
627	39 31 29	25.039 9681	.001 594 896
628	39 43 84	25.059 9282	.001 592 357
629	39 56 41	25.079 8724	.001 589 825
630	39 69 00	25.099 8008	.001 587 302
631	39 81 61	25.119 7134	.001 584 786
632	39 94 24	25.139 6102	.001 582 278
633	40 06 89	25.159 4913	.001 579 779
634	40 19 56	25.179 3566	.001 577 287
635	40 32 25	25.199 2063	.001 574 803
636	40 44 96	25.219 0404	.001 572 327
637	40 57 69	25.238 8589	.001 569 859
638	40 70 44	25.258 6619	.001 567 398
639	40 83 21	25.278 4493	.001 564 945
640	40 96 00	25.298 2213	.001 562 500
641	41 08 81	25.317 9778	.001 560 062
642	41 21 64	25.337 7189	.001 557 632
643	41 34 49	25.357 4447	.001 555 210
644	41 47 36	25.377 1551	.001 552 795
645	41 60 25	25.396 8502	.001 550 388
646	41 73 16	25.416 5301	.001 547 988
647	41 86 09	25.436 1947	.001 545 595
648	41 99 04	25.455 8441	.001 543 210
649	42 12 01	25.475 4784	.001 540 832
650	42 25 00	25.495 0976	.001 538 462

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
651	42 38 01	25.514 7016	.001 536 098
652	42 51 04	25.534 2907	.001 533 742
653	42 64 09	25.553 8647	.001 531 394
654	42 77 16	25.573 4237	.001 529 052
655	42 90 25	25.592 9678	.001 526 718
656	43 03 36	25.612 4969	.001 524 390
657	43 16 49	25.632 0112	.001 522 070
658	43 29 64	25.651 5107	.001 519 757
659	43 42 81	25.670 9953	.001 517 451
660	43 56 00	25.690 4652	.001 515 152
661	43 69 21	25.709 9203	.001 512 859
662	43 82 44	25.729 3607	.001 510 574
663	43 95 69	25.748 7864	.001 508 296
664	44 08 96	25.768 1975	.001 506 024
665	44 22 25	25.787 5939	.001 503 759
666	44 35 56	25.806 9758	.001 501 502
667	44 48 89	25.826 3431	.001 499 250
668	44 62 24	25.845 6960	.001 497 006
669	44 75 61	25.865 0343	.001 494 768
670	44 89 00	25.884 3582	.001 492 537
671	45 02 41	25.903 6677	.001 490 313
672	45 15 84	25.922 9628	.001 488 095
673	45 29 29	25.942 2435	.001 485 884
674	45 42 76	25.961 5100	.001 483 680
675	45 56 25	25.980 7621	.001 481 481
676	45 69 76	26.000 0000	.001 479 290
677	45 83 29	26.019 2237	.001 477 105
678	45 96 84	26.038 4331	.001 474 926
679	46 10 41	26.057 6284	.001 472 754
680	46 24 00	26.076 8096	.001 470 588
681	46 37 61	26.095 9767	.001 468 429
682	46 51 24	26.115 1297	.001 466 276
683	46 64 89	26.134 2687	.001 464 129
684	46 78 56	26.153 3937	.001 461 988
685	46 92 25	26.172 5047	.001 459 854
686	47 05 96	26.191 6017	.001 457 726
687	47 19 69	26.210 6848	.001 455 604
688	47 33 44	26.229 7541	.001 453 488
689	47 47 21	26.248 8095	.001 451 379
690	47 61 00	26.267 8511	.001 449 275
691	47 74 81	26.286 8789	.001 447 178
692	47 88 64	26.305 8929	.001 445 087
693	48 02 49	26.324 8932	.001 443 001
694	48 16 36	26.343 8797	.001 440 922
695	48 30 25	26.362 8527	.001 438 849
696	48 44 16	26.381 8119	.001 436 782
697	48 58 09	26.400 7576	.001 434 720
698	48 72 04	26.419 6896	.001 432 665
699	48 86 01	26.438 6081	.001 430 615
700	49 00 00	26.457 5131	.001 428 571

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
701	49 14 01	26.476 4046	.001 426 534
702	49 28 04	26.495 2826	.001 424 501
703	49 42 09	26.514 1472	.001 422 475
704	49 56 16	26.532 9983	.001 420 455
705	49 70 25	26.551 8361	.001 418 440
706	49 84 36	26.570 6605	.001 416 431
707	49 98 49	26.589 4716	.001 414 427
708	50 12 64	26.608 2694	.001 412 429
709	50 26 81	26.627 0539	.001 410 437
710	50 41 00	26.645 8252	.001 408 451
711	50 55 21	26.664 5833	.001 406 470
712	50 69 44	26.683 3281	.001 404 494
713	50 83 69	26.702 0598	.001 402 525
714	50 97 96	26.720 7784	.001 400 560
715	51 12 25	26.739 4839	.001 398 601
716	51 26 56	26.758 1763	.001 396 648
717	51 40 89	26.776 8557	.001 394 700
718	51 55 24	26.795 5220	.001 392 758
719	51 69 61	26.814 1754	.001 390 821
720	51 84 00	26.832 8157	.001 388 889
721	51 98 41	26.851 4432	.001 386 963
722	52 12 84	26.870 0577	.001 385 042
723	52 27 29	26.888 6593	.001 383 126
724	52 41 76	26.907 2481	.001 381 215
725	52 56 25	26.925 8240	.001 379 310
726	52 70 76	26.944 3872	.001 377 410
727	52 85 29	26.962 9375	.001 375 516
728	52 99 84	26.981 4751	.001 373 626
729	53 14 41	27.000 0000	.001 371 742
730	53 29 00	27.018 5122	.001 369 863
731	53 43 61	27.037 0117	.001 367 989
732	53 58 24	27.055 4985	.001 366 120
733	53 72 89	27.073 9727	.001 364 256
734	53 87 56	27.092 4344	.001 362 398
735	54 02 25	27.110 8834	.001 360 544
736	54 16 96	27.129 3199	.001 358 696
737	54 31 69	27.147 7439	.001 356 852
738	54 46 44	27.166 1554	.001 355 014
739	54 61 21	27.184 5544	.001 353 180
740	54 76 00	27.202 9410	.001 351 351
741	54 90 81	27.221 3152	.001 349 528
742	55 05 64	27.239 6769	.001 347 709
743	55 20 49	27.258 0263	.001 345 895
744	55 35 36	27.276 3634	.001 344 086
745	55 50 25	27.294 6881	.001 342 282
746	55 65 16	27.313 0006	.001 340 483
747	55 80 09	27.331 3007	.001 338 688
748	55 95 04	27.349 5887	.001 336 898
749	56 10 01	27.367 8644	.001 335 113
750	56 25 00	27.386 1279	.001 333 333

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
751	56 40 01	27.404 3792	.001 331 558
752	56 55 04	27.422 6184	.001 329 787
753	56 70 09	27.440 8455	.001 328 021
754	56 85 16	27.459 0604	.001 326 260
755	57 00 25	27.477 2633	.001 324 503
756	57 15 36	27.495 4542	.001 322 751
757	57 30 49	27.513 6330	.001 321 004
758	57 45 64	27.531 7998	.001 319 261
759	57 60 81	27.549 9546	.001 317 523
760	57 76 00	27.568 0975	.001 315 789
761	57 91 21	27.586 2284	.001 314 060
762	58 06 44	27.604 3475	.001 312 336
763	58 21 69	27.622 4546	.001 310 616
764	58 36 96	27.640 5499	.001 308 901
765	58 52 25	27.658 6334	.001 307 190
766	58 67 56	27.676 7050	.001 305 483
767	58 82 89	27.694 7648	.001 303 781
768	58 98 24	27.712 8129	.001 302 083
769	59 13 61	27.730 8492	.001 300 390
770	59 29 00	27.748 8739	.001 298 701
771	59 44 41	27.766 8868	.001 297 017
772	59 59 84	27.784 8880	.001 295 337
773	59 75 29	27.802 8775	.001 293 661
774	59 90 76	27.820 8555	.001 291 990
775	60 06 25	27.838 8218	.001 290 323
776	60 21 76	27.856 7766	.001 288 660
777	60 37 29	27.874 7197	.001 287 001
778	60 52 84	27.892 6514	.001 285 347
779	60 68 41	27.910 5715	.001 283 697
780	60 84 00	27.928 4801	.001 282 051
781	60 99 61	27.946 3772	.001 280 410
782	61 15 24	27.964 2629	.001 278 772
783	61 30 89	27.982 1372	.001 277 139
784	61 46 56	28.000 0000	.001 275 510
785	61 62 25	28.017 8515	.001 273 885
786	61 77 96	28.035 6915	.001 272 265
787	61 93 69	28.053 5203	.001 270 648
788	62 09 44	28.071 3377	.001 269 036
789	62 25 21	28.089 1438	.001 267 427
790	62 41 00	28.106 9386	.001 265 823
791	62 56 81	28.124 7222	.001 264 223
792	62 72 64	28.142 4946	.001 262 626
793	62 88 49	28.160 2557	.001 261 034
794	63 04 36	28.178 0056	.001 259 446
795	63 20 25	28.195 7444	.001 257 862
796	63 36 16	28.213 4720	.001 256 281
797	63 52 09	28.231 1884	.001 254 705
798	63 68 04	28.248 8938	.001 253 133
799	63 84 01	28.266 5881	.001 251 564
800	64 00 00	28.284 2712	.001 250 000

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
801	64 16 01	28.301 9434	.001 248 439
802	64 32 04	28.319 6045	.001 246 883
803	64 48 09	28.337 2546	.001 245 330
804	64 64 16	28.354 8938	.001 243 781
805	64 80 25	28.372 5219	.001 242 236
806	64 96 36	28.390 1391	.001 240 695
807	65 12 49	28.407 7454	.001 239 157
808	65 28 64	28.425 3408	.001 237 624
809	65 44 81	28.442 9253	.001 236 094
810	65 61 00	28.460 4989	.001 234 568
811	65 77 21	28.478 0617	.001 233 046
812	65 93 44	28.495 6137	.001 231 527
813	66 09 69	28.513 1549	.001 230 012
814	66 25 96	28.530 6852	.001 228 501
815	66 42 25	28.548 2048	.001 226 994
816	66 58 56	28.565 7137	.001 225 490
817	66 74 89	28.583 2119	.001 223 990
818	66 91 24	28.600 6993	.001 222 494
819	67 07 61	28.618 1760	.001 221 001
820	67 24 00	28.635 6421	.001 219 512
821	67 40 41	28.653 0976	.001 218 027
822	67 56 84	28.670 5424	.001 216 545
823	67 73 29	28.687 9766	.001 215 067
824	67 89 76	28.705 4002	.001 213 592
825	68 06 25	28.722 8132	.001 212 121
826	68 22 76	28.740 2157	.001 210 654
827	68 39 29	28.757 6077	.001 209 190
828	68 55 84	28.774 9891	.001 207 729
829	68 72 41	28.792 3601	.001 206 273
830	68 89 00	28.809 7206	.001 204 819
831	69 05 61	28.827 0706	.001 203 369
832	69 22 24	28.844 4102	.001 201 923
833	69 38 89	28.861 7394	.001 200 480
834	69 55 56	28.879 0582	.001 199 041
835	69 72 25	28.896 3666	.001 197 605
836	69 88 96	28.913 6646	.001 196 172
837	70 05 69	28.930 9523	.001 194 743
838	70 22 44	28.948 2297	.001 193 317
839	70 39 21	28.965 4967	.001 191 895
840	70 56 00	28.982 7535	.001 190 476
841	70 72 81	29.000 0000	.001 189 061
842	70 89 64	29.017 2363	.001 187 648
843	71 06 49	29.034 4623	.001 186 240
844	71 23 36	29.051 6781	.001 184 834
845	71 40 25	29.068 8837	.001 183 432
846	71 57 16	29.086 0791	.001 182 033
847	71 74 09	29.103 2644	.001 180 638
848	71 91 04	29.120 4396	.001 179 245
849	72 08 01	29.137 6046	.001 177 856
850	72 25 00	29.154 7595	.001 176 471

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
851	72 42 01	29.171 9043	.001 175 088
852	72 59 04	29.189 0390	.001 173 709
853	72 76 09	29.206 1637	.001 172 333
854	72 93 16	29.223 2784	.001 170 960
855	73 10 25	29.240 3830	.001 169 591
856	73 27 36	29.257 4777	.001 168 224
857	73 44 49	29.274 5623	.001 166 861
858	73 61 64	29.291 6370	.001 165 501
859	73 78 81	29.308 7018	.001 164 144
860	73 96 00	29.325 7566	.001 162 791
861	74 13 21	29.342 8015	.001 161 440
862	74 30 44	29.359 8365	.001 160 093
863	74 47 69	29.376 8616	.001 158 749
864	74 64 96	29.393 8769	.001 157 407
865	74 82 25	29.410 8823	.001 156 069
866	74 99 56	29.427 8779	.001 154 734
867	75 16 89	29.444 8637	.001 153 403
868	75 34 24	29.461 8397	.001 152 074
869	75 51 61	29.478 8059	.001 150 748
870	75 69 00	29.495 7624	.001 149 425
871	75 86 41	29.512 7091	.001 148 106
872	76 03 84	29.529 6461	.001 146 789
873	76 21 29	29.546 5734	.001 145 475
874	76 38 76	29.563 4910	.001 144 165
875	76 56 25	29.580 3989	.001 142 857
876	76 73 76	29.597 2972	.001 141 553
877	76 91 29	29.614 1858	.001 140 251
878	77 08 84	29.631 0648	.001 138 952
879	77 26 41	29.647 9342	.001 137 656
880	77 44 00	29.664 7939	.001 136 364
881	77 61 61	29.681 6442	.001 135 074
882	77 79 24	29.698 4848	.001 133 787
883	77 96 89	29.715 3159	.001 132 503
884	78 14 56	29.732 1375	.001 131 222
885	78 32 25	29.748 9496	.001 129 944
886	78 49 96	29.765 7521	.001 128 668
887	78 67 69	29.782 5452	.001 127 396
888	78 85 44	29.799 3289	.001 126 126
889	79 03 21	29.816 1030	.001 124 859
890	79 21 00	29.832 8678	.001 123 596
891	79 38 81	29.849 6231	.001 122 334
892	79 56 64	29.866 3690	.001 121 076
893	79 74 49	29.883 1056	.001 119 821
894	79 92 36	29.899 8328	.001 118 568
895	80 10 25	29.916 5506	.001 117 318
896	80 28 16	29.932 2591	.001 116 071
897	80 46 09	29.949 9583	.001 114 827
898	80 64 04	29.966 6481	.001 113 586
899	80 82 01	29.983 3287	.001 112 347
900	81 00 00	30.000 0000	.001 111 111

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
901	81 18 01	30.016 6620	.001 109 878
902	81 36 04	30.033 3148	.001 108 647
903	81 54 09	30.049 9584	.001 107 420
904	81 72 16	30.066 5928	.001 106 195
905	81 90 25	30.083 2179	.001 104 972
906	82 08 36	30.099 8339	.001 103 753
907	82 26 49	30.116 4407	.001 102 536
908	82 44 64	30.133 0383	.001 101 322
909	82 62 81	30.149 6269	.001 100 110
910	82 81 00	30.166 2063	.001 098 901
911	82 99 21	30.182 7765	.001 097 695
912	83 17 44	30.199 3377	.001 096 491
913	83 35 69	30.215 8899	.001 095 290
914	83 53 96	30.232 4329	.001 094 092
915	83 72 25	30.248 9669	.001 092 896
916	83 90 56	30.265 4919	.001 091 703
917	84 08 89	30.282 0079	.001 090 513
918	84 27 24	30.298 5148	.001 089 325
919	84 45 61	30.315 0128	.001 088 139
920	84 64 00	30.331 5018	.001 086 957
921	84 82 41	30.347 9818	.001 085 776
922	85 00 84	30.364 4529	.001 084 599
923	85 19 29	30.380 9151	.001 083 424
924	85 37 76	30.397 3683	.001 082 251
925	85 56 25	30.413 8127	.001 081 081
926	85 74 76	30.430 2481	.001 079 914
927	85 93 29	30.446 6747	.001 078 749
928	86 11 84	30.463 0924	.001 077 586
929	86 30 41	30.479 5013	.001 076 426
930	86 49 00	30.495 9014	.001 075 269
931	86 67 61	30.512 2926	.001 074 114
932	86 86 24	30.528 6750	.001 072 961
933	87 04 89	30.545 0487	.001 071 811
934	87 23 56	30.561 4136	.001 070 664
935	87 42 25	30.577 7697	.001 069 519
936	87 60 96	30.594 1171	.001 068 376
937	87 79 69	30.610 4557	.001 067 236
938	87 98 44	30.626 7857	.001 066 098
939	88 17 21	30.643 1069	.001 064 963
940	88 36 00	30.659 4194	.001 063 830
941	88 54 81	30.675 7233	.001 062 699
942	88 73 64	30.692 0185	.001 061 571
943	88 92 49	30.708 3051	.001 060 445
944	89 11 36	30.724 5830	.001 059 322
945	89 30 25	30.740 8523	.001 058 201
946	89 49 16	30.757 1130	.001 057 082
947	89 68 09	30.773 3651	.001 055 966
948	89 87 04	30.789 6086	.001 054 852
949	90 06 01	30.805 8436	.001 053 741
950	90 25 00	30.822 0700	.001 052 632

TABLE B — *Continued*

SQUARES, SQUARE ROOTS AND RECIPROCALS OF THE NATURAL NUMBERS FROM 1 TO 1000

n	n^2	$n^{1/2}$	$1/n$
951	90 44 01	30.838 2879	.001 051 525
952	90 63 04	30.854 4972	.001 050 420
953	90 82 09	30.870 6981	.001 049 318
954	91 01 16	30.886 8904	.001 048 218
955	91 20 25	30.903 0743	.001 047 120
956	91 39 36	30.919 2497	.001 046 025
957	91 58 49	30.935 4166	.001 044 932
958	91 77 64	30.951 5751	.001 043 841
959	91 96 81	30.967 7251	.001 042 753
960	92 16 00	30.983 8668	.001 041 667
961	92 35 21	31.000 0000	.001 040 583
962	92 54 44	31.016 1248	.001 039 501
963	92 73 69	31.032 2413	.001 038 422
964	92 92 96	31.048 3494	.001 037 344
965	93 12 25	31.064 4491	.001 036 269
966	93 31 56	31.080 5405	.001 035 197
967	93 50 89	31.096 6236	.001 034 126
968	93 70 24	31.112 6984	.001 033 058
969	93 89 61	31.128 7648	.001 031 992
970	94 09 00	31.144 8230	.001 030 928
971	94 28 41	31.160 8729	.001 029 866
972	94 47 84	31.176 9145	.001 028 807
973	94 67 29	31.192 9479	.001 027 749
974	94 86 76	31.208 9731	.001 026 694
975	95 06 25	31.224 9900	.001 025 641
976	95 25 76	31.240 9987	.001 024 590
977	95 45 29	31.256 9992	.001 023 541
978	95 64 84	31.272 9915	.001 022 495
979	95 84 41	31.288 9757	.001 021 450
980	96 04 00	31.304 9517	.001 020 408
981	96 23 61	31.320 9195	.001 019 368
982	96 43 24	31.336 8792	.001 018 330
983	96 62 89	31.352 8308	.001 017 294
984	96 82 56	31.368 7743	.001 016 260
985	97 02 25	31.384 7097	.001 015 228
986	97 21 96	31.400 6369	.001 014 199
987	97 41 69	31.416 5561	.001 013 171
988	97 61 44	31.432 4673	.001 012 146
989	97 81 21	31.448 3704	.001 011 122
990	98 01 00	31.464 2654	.001 010 101
991	98 20 81	31.480 1525	.001 009 082
992	98 40 64	31.496 0315	.001 008 065
993	98 60 49	31.511 9025	.001 007 049
994	98 80 36	31.527 7655	.001 006 036
995	99 00 25	31.543 6206	.001 005 025
996	99 20 16	31.559 4677	.001 004 016
997	99 40 09	31.575 3068	.001 003 009
998	99 60 04	31.591 1380	.001 002 004
999	99 80 01	31.606 9613	.001 001 001
1000	1 00 00 00	31.622 7766	.001 000 000

TABLE C

First Six Powers of the Natural Numbers from 1 to 50

n	n^2	n^3	n^4	n^5	n^6	n
1	1	1	1	1	1	1
2	4	8	16	32	64	2
3	9	27	81	243	729	3
4	16	64	256	1 024	4 096	4
5	25	125	625	3 125	15 625	5
6	36	216	1 296	7 776	46 656	6
7	49	343	2 401	16 807	117 649	7
8	64	512	4 096	32 768	262 144	8
9	81	729	6 561	59 049	531 441	9
10	100	1 000	10 000	100 000	1 000 000	10
11	121	1 331	14 641	161 051	1 771 561	11
12	144	1 728	20 736	248 832	2 985 984	12
13	169	2 197	28 561	371 293	4 826 809	13
14	196	2 744	38 416	537 824	7 529 536	14
15	225	3 375	50 625	759 375	11 390 625	15
16	256	4 096	65 536	1 048 576	16 777 216	16
17	289	4 913	83 521	1 419 857	24 137 569	17
18	324	5 832	104 976	1 889 568	34 012 224	18
19	361	6 859	130 321	2 476 099	47 045 881	19
20	400	8 000	160 000	3 200 000	64 000 000	20
21	441	9 261	194 481	4 084 101	85 766 121	21
22	484	10 648	234 256	5 153 632	113 379 904	22
23	529	12 167	279 841	6 436 343	148 035 889	23
24	576	13 824	331 776	7 962 624	191 102 976	24
25	625	15 625	390 625	9 765 625	244 140 625	25
26	676	17 576	456 976	11 881 376	308 915 776	26
27	729	19 683	531 441	14 348 907	387 420 489	27
28	784	21 952	614 656	17 210 368	481 890 304	28
29	841	24 389	707 281	20 511 149	594 823 321	29
30	900	27 000	810 000	24 300 000	729 000 000	30
31	961	29 791	923 521	28 629 151	887 503 681	31
32	1 024	32 768	1 048 576	33 554 432	1 073 741 824	32
33	1 089	35 937	1 185 921	39 135 393	1 291 467 969	33
34	1 156	39 304	1 336 336	45 435 424	1 544 804 416	34
35	1 225	42 875	1 500 625	52 521 875	1 838 265 625	35
36	1 296	46 656	1 679 616	60 466 176	2 176 782 336	36
37	1 369	50 653	1 874 161	69 343 957	2 565 726 409	37
38	1 444	54 872	2 085 136	79 235 168	3 010 936 384	38
39	1 521	59 319	2 313 441	90 224 199	3 518 743 761	39
40	1 600	64 000	2 560 000	102 400 000	4 096 000 000	40
41	1 681	68 921	2 825 761	115 856 201	4 750 104 241	41
42	1 764	74 088	3 111 696	130 691 232	5 489 031 744	42
43	1 849	79 507	3 418 801	147 008 443	6 321 363 049	43
44	1 936	85 184	3 748 096	164 916 224	7 256 313 856	44
45	2 025	91 125	4 100 625	184 528 125	8 303 765 625	45
46	2 116	97 336	4 477 456	205 962 976	9 474 296 896	46
47	2 209	103 823	4 879 681	229 345 007	10 779 215 329	47
48	2 304	110 592	5 308 416	254 803 968	12 230 590 464	48
49	2 401	117 649	5 764 801	282 475 249	13 841 287 201	49
50	2 500	125 000	6 250 000	312 500 000	15 625 000 000	50

TABLE D

Sums of the First Six Powers of the Natural Numbers from 1 to 50

n	$\Sigma(n)$	$\Sigma(n^2)$	$\Sigma(n^3)$	$\Sigma(n^4)$	$\Sigma(n^5)$	$\Sigma(n^6)$
1	1	1	1	1	1	1
2	3	5	9	17	33	65
3	6	14	36	98	276	794
4	10	30	100	354	1 300	4 890
5	15	55	225	979	4 425	20 515
6	21	91	441	2 275	12 201	67 171
7	28	140	784	4 676	29 008	184 820
8	36	204	1 296	8 772	61 776	446 964
9	45	285	2 025	15 333	120 825	978 405
10	55	385	3 025	25 333	220 825	1 978 405
11	66	506	4 356	39 974	381 876	3 749 966
12	78	650	6 084	60 710	630 708	6 735 950
13	91	819	8 281	89 271	1 002 001	11 562 759
14	105	1 015	11 025	127 687	1 539 825	19 092 295
15	120	1 240	14 400	178 312	2 299 200	30 482 920
16	136	1 496	18 496	234 848	3 347 776	47 260 136
17	153	1 785	23 409	327 369	4 767 633	71 397 705
18	171	2 109	29 241	432 345	6 657 201	105 409 929
19	190	2 470	36 100	562 666	9 133 300	152 455 810
20	210	2 870	44 100	722 666	12 333 300	216 455 810
21	231	3 311	53 361	917 147	16 417 401	302 221 931
22	253	3 795	64 009	1 151 403	21 571 033	415 601 835
23	276	4 324	76 176	1 431 244	28 007 376	563 637 724
24	300	4 900	90 000	1 763 020	35 970 000	754 740 700
25	325	5 525	105 625	2 153 645	45 735 625	998 881 325
26	351	6 201	123 201	2 610 621	57 617 001	1 307 797 101
27	378	6 930	142 884	3 142 062	71 965 908	1 695 217 590
28	406	7 714	164 836	3 756 718	89 176 276	2 177 107 894
29	435	8 555	189 225	4 463 999	109 687 425	2 771 931 215
30	465	9 455	216 225	5 273 999	133 987 425	3 500 931 215
31	496	10 416	246 016	6 197 520	162 616 576	4 388 434 896
32	528	11 440	278 784	7 246 096	196 171 008	5 462 176 720
33	561	12 529	314 721	8 432 017	235 306 401	6 753 644 689
34	595	13 685	354 025	9 768 353	280 741 825	8 298 449 105
35	630	14 910	396 900	11 268 978	333 263 700	10 136 714 730
36	666	16 206	443 556	12 948 594	393 729 876	12 313 497 066
37	703	17 575	494 209	14 822 755	463 073 833	14 879 223 475
38	741	19 019	549 081	16 907 891	542 309 001	17 890 159 859
39	780	20 540	608 400	19 221 332	632 533 200	21 408 903 620
40	820	22 140	672 400	21 781 332	734 933 200	25 504 903 620
41	861	23 821	741 321	24 607 093	850 789 401	30 255 007 861
42	903	25 585	815 409	27 718 789	981 480 633	35 744 039 605
43	946	27 434	894 916	31 137 590	1 128 489 076	42 065 402 654
44	990	29 370	980 100	34 885 686	1 293 405 300	49 321 716 510
45	1 035	31 395	1 071 225	38 986 311	1 477 933 425	57 625 482 135
46	1 081	33 511	1 168 561	43 463 767	1 683 896 401	67 099 779 031
47	1 128	35 720	1 272 384	48 343 448	1 913 241 408	77 878 994 360
48	1 176	38 024	1 382 976	53 651 864	2 168 045 376	90 109 584 824
49	1 225	40 425	1 500 625	59 416 665	2 450 520 625	103 950 872 025
50	1 275	42 925	1 625 625	65 666 665	2 763 020 625	119 575 872 025

TABLE E

Sums of the First Six Powers of the First Fifty Odd Natural Numbers

t	${}_0S_1$	${}_0S_2$	${}_0S_3$	${}_0S_4$	${}_0S_5$	${}_0S_6$	t
1	1	1	1	1	1	1	1
2	4	10	28	82	244	730	2
3	9	35	153	707	3 369	16 355	3
4	16	84	496	3 108	20 176	134 004	4
5	25	165	1 225	9 669	79 225	665 445	5
6	36	286	2 556	24 310	240 276	2 437 006	6
7	49	455	4 753	52 871	611 569	7 263 815	7
8	64	680	8 128	103 496	1 370 944	18 654 440	8
9	81	969	13 041	187 017	2 790 801	42 792 009	9
10	100	1 330	19 900	317 338	5 266 900	89 837 890	10
11	121	1 771	29 161	511 819	9 351 001	175 604 011	11
12	144	2 300	41 328	791 660	15 787 344	323 639 900	12
13	169	2 925	56 953	1 182 285	25 552 969	567 780 525	13
14	196	3 654	76 636	1 713 726	39 901 876	955 201 014	14
15	225	4 495	101 025	2 421 007	60 413 025	1 550 024 335	15
16	256	5 456	130 816	3 344 528	89 042 176	2 437 528 016	16
17	289	6 545	166 753	4 530 449	128 177 569	3 728 995 985	17
18	324	7 770	209 628	6 031 074	180 699 444	5 567 261 610	18
19	361	9 139	260 281	7 905 235	250 043 401	8 132 988 019	19
20	400	10 660	319 600	10 218 676	340 267 600	11 651 731 780	20
21	441	12 341	388 521	13 044 437	456 123 801	16 401 836 021	21
22	484	14 190	468 028	16 463 238	603 132 244	22 723 199 070	22
23	529	16 215	559 153	20 563 863	787 660 369	31 026 964 695	23
24	576	18 424	662 976	25 443 544	1 017 005 376	41 806 180 024	24
25	625	20 825	780 625	31 208 345	1 299 480 625	55 647 467 225	25
26	676	23 426	913 276	37 973 546	1 644 505 876	73 243 755 026	26
27	729	26 235	1 062 153	45 864 027	2 062 701 369	95 408 116 155	27
28	784	29 260	1 228 528	55 014 652	2 565 985 744	123 088 756 780	28
29	841	32 509	1 413 721	65 570 653	3 167 677 801	157 385 204 029	29
30	900	35 990	1 619 100	77 688 014	3 882 602 100	199 565 737 670	30
31	961	39 711	1 846 081	91 533 855	4 727 198 401	251 086 112 031	31
32	1 024	43 680	2 096 128	107 286 816	5 719 634 944	313 609 614 240	32
33	1 089	47 905	2 370 753	125 137 441	6 879 925 569	389 028 504 865	33
34	1 156	52 394	2 671 516	145 288 562	8 230 050 676	479 486 887 034	34
35	1 225	57 155	3 000 025	167 955 683	9 794 082 025	587 405 050 115	35
36	1 296	62 196	3 357 936	193 367 364	11 598 311 376	715 505 334 036	36
37	1 369	67 525	3 746 953	221 765 605	13 671 382 969	866 839 560 325	37
38	1 444	73 150	4 168 828	253 406 230	16 044 429 844	1 044 818 075 950	38
39	1 521	79 079	4 625 361	288 559 271	18 751 214 001	1 253 240 456 039	39
40	1 600	85 320	5 118 400	327 509 352	21 828 270 400	1 496 327 911 560	40
41	1 681	91 881	5 649 841	370 556 073	25 315 054 801	1 778 757 448 041	41
42	1 764	98 770	6 221 628	418 014 394	29 254 095 444	2 105 697 821 410	42
43	1 849	105 995	6 835 753	470 215 019	33 691 148 569	2 482 847 337 035	43
44	1 936	113 564	7 494 256	527 504 780	38 675 357 776	2 916 473 538 044	44
45	2 025	121 485	8 199 225	590 247 021	44 259 417 225	3 413 454 829 005	45
46	2 116	129 766	8 952 796	658 821 982	50 499 738 676	3 981 324 081 046	46
47	2 209	138 415	9 757 153	733 627 183	57 456 622 369	4 628 314 264 495	47
48	2 304	147 440	10 614 528	815 077 808	65 194 431 744	5 363 406 155 120	48
49	2 401	156 849	11 527 201	903 607 089	73 781 772 001	6 196 378 160 049	49
50	2 500	166 650	12 497 500	999 666 690	83 291 672 500	7 137 858 309 450	50

TABLE F

Areas and Ordinates of the Normal Curve of Error in Terms of Abscissa

x/σ	Area between maximum ordinate and ordinate at x/σ	Ordinate at x/σ (z)	x/σ	Area between maximum ordinate and ordinate at x/σ	Ordinate at x/σ (z)
.00	.00000	.39894	.50	.19146	.35207
.01	.00399	.39892	.51	.19497	.35029
.02	.00798	.39886	.52	.19847	.34849
.03	.01197	.39876	.53	.20194	.34667
.04	.01595	.39862	.54	.20540	.34482
.05	.01994	.39844	.55	.20884	.34294
.06	.02392	.39822	.56	.21226	.34105
.07	.02790	.39797	.57	.21566	.33912
.08	.03188	.39767	.58	.21904	.33718
.09	.03586	.39733	.59	.22240	.33521
.10	.03983	.39695	.60	.22575	.33322
.11	.04380	.39654	.61	.22907	.33121
.12	.04776	.39608	.62	.23237	.32918
.13	.05172	.39559	.63	.23565	.32713
.14	.05567	.39505	.64	.23891	.32506
.15	.05962	.39448	.65	.24215	.32297
.16	.06356	.39387	.66	.24537	.32086
.17	.06749	.39322	.67	.24857	.31874
.18	.07142	.39253	.68	.25175	.31659
.19	.07535	.39181	.69	.25490	.31443
.20	.07926	.39104	.70	.25804	.31225
.21	.08317	.39024	.71	.26115	.31006
.22	.08706	.38940	.72	.26424	.30785
.23	.09095	.38853	.73	.26730	.30563
.24	.09483	.38762	.74	.27035	.30339
.25	.09871	.38667	.75	.27337	.30114
.26	.10257	.38568	.76	.27637	.29887
.27	.10642	.38466	.77	.27935	.29659
.28	.11026	.38361	.78	.28230	.29431
.29	.11409	.38251	.79	.28524	.29200
.30	.11791	.38139	.80	.28814	.28969
.31	.12172	.38023	.81	.29103	.28737
.32	.12552	.37903	.82	.29389	.28504
.33	.12930	.37780	.83	.29673	.28269
.34	.13307	.37654	.84	.29955	.28034
.35	.13683	.37524	.85	.30234	.27798
.36	.14058	.37391	.86	.30511	.27562
.37	.14431	.37255	.87	.30785	.27324
.38	.14803	.37115	.88	.31057	.27086
.39	.15173	.36973	.89	.31327	.26848
.40	.15542	.36827	.90	.31594	.26609
.41	.15910	.36678	.91	.31859	.26369
.42	.16276	.36526	.92	.32121	.26129
.43	.16640	.36371	.93	.32381	.25888
.44	.17003	.36213	.94	.32639	.25647
.45	.17364	.36053	.95	.32894	.25406
.46	.17724	.35889	.96	.33147	.25164
.47	.18082	.35723	.97	.33398	.24923
.48	.18439	.35553	.98	.33646	.24681
.49	.18793	.35381	.99	.33891	.24439

TABLE F — *Continued*

AREAS AND ORDINATES OF THE NORMAL CURVE OF ERROR IN TERMS OF ABSCISSA

$x \div \sigma$	Area between maximum ordinate and ordinate at $x \div \sigma$	Ordinate at $x \div \sigma$ (z)	$x \div \sigma$	Area between maximum ordinate and ordinate at $x \div \sigma$	Ordinate at $x \div \sigma$ (z)
1.00	.84134	.24197	1.50	.48819	.12952
1.01	.84375	.23955	1.51	.49448	.12758
1.02	.84614	.23713	1.52	.49574	.12566
1.03	.84850	.23471	1.53	.49699	.12376
1.04	.85083	.23230	1.54	.49822	.12188
1.05	.85314	.22988	1.55	.49943	.12001
1.06	.85543	.22747	1.56	.50062	.11816
1.07	.85769	.22506	1.57	.50179	.11632
1.08	.85993	.22265	1.58	.50295	.11450
1.09	.86214	.22025	1.59	.50408	.11270
1.10	.86433	.21785	1.60	.50520	.11092
1.11	.86650	.21546	1.61	.50630	.10915
1.12	.86864	.21307	1.62	.50738	.10741
1.13	.87076	.21069	1.63	.50845	.10567
1.14	.87286	.20831	1.64	.50950	.10396
1.15	.87493	.20594	1.65	.51053	.10226
1.16	.87698	.20357	1.66	.51154	.10059
1.17	.87900	.20121	1.67	.51254	.09893
1.18	.88100	.19886	1.68	.51352	.09728
1.19	.88298	.19652	1.69	.51449	.09566
1.20	.88493	.19419	1.70	.51543	.09405
1.21	.88686	.19186	1.71	.51637	.09246
1.22	.88877	.18954	1.72	.51728	.09089
1.23	.89065	.18724	1.73	.51818	.08933
1.24	.89251	.18494	1.74	.51907	.08780
1.25	.89435	.18265	1.75	.51994	.08628
1.26	.89617	.18037	1.76	.52080	.08478
1.27	.89796	.17810	1.77	.52164	.08329
1.28	.89973	.17585	1.78	.52246	.08183
1.29	.90147	.17360	1.79	.52327	.08038
1.30	.90320	.17137	1.80	.52407	.07895
1.31	.90490	.16915	1.81	.52485	.07754
1.32	.90658	.16694	1.82	.52562	.07614
1.33	.90824	.16474	1.83	.52638	.07477
1.34	.90988	.16256	1.84	.52712	.07341
1.35	.91149	.16038	1.85	.52784	.07206
1.36	.91309	.15822	1.86	.52856	.07074
1.37	.91466	.15608	1.87	.52926	.06943
1.38	.91621	.15395	1.88	.52995	.06814
1.39	.91774	.15183	1.89	.53062	.06687
1.40	.91924	.14973	1.90	.53128	.06562
1.41	.92073	.14764	1.91	.53193	.06438
1.42	.92220	.14556	1.92	.53257	.06316
1.43	.92364	.14350	1.93	.53320	.06195
1.44	.92507	.14146	1.94	.53381	.06077
1.45	.92647	.13943	1.95	.53441	.05959
1.46	.92786	.13742	1.96	.53500	.05844
1.47	.92922	.13542	1.97	.53558	.05730
1.48	.93056	.13344	1.98	.53615	.05618
1.49	.93189	.13147	1.99	.53670	.05508

TABLE F — *Continued*

AREAS AND ORDINATES OF THE NORMAL CURVE OF ERROR IN TERMS OF ABSCISSA

x/σ	Area between maximum ordinate and ordinate at x/σ	Ordinate at x/σ (z)	x/σ	Area between maximum ordinate and ordinate at x/σ	Ordinate at x/σ (z)
2.00	.47725	.05399	2.50	.49379	.01753
2.01	.47778	.05292	2.51	.49396	.01709
2.02	.47831	.05186	2.52	.49413	.01667
2.03	.47882	.05082	2.53	.49430	.01625
2.04	.47932	.04980	2.54	.49446	.01585
2.05	.47982	.04879	2.55	.49461	.01545
2.06	.48030	.04780	2.56	.49477	.01506
2.07	.48077	.04682	2.57	.49492	.01468
2.08	.48124	.04586	2.58	.49506	.01431
2.09	.48169	.04491	2.59	.49520	.01394
2.10	.48214	.04398	2.60	.49534	.01358
2.11	.48257	.04307	2.61	.49547	.01323
2.12	.48300	.04217	2.62	.49560	.01289
2.13	.48341	.04128	2.63	.49573	.01256
2.14	.48382	.04041	2.64	.49585	.01223
2.15	.48422	.03955	2.65	.49598	.01191
2.16	.48461	.03871	2.66	.49609	.01160
2.17	.48500	.03788	2.67	.49621	.01130
2.18	.48537	.03706	2.68	.49632	.01100
2.19	.48574	.03626	2.69	.49643	.01071
2.20	.48610	.03547	2.70	.49653	.01042
2.21	.48645	.03470	2.71	.49664	.01014
2.22	.48679	.03394	2.72	.49674	.00987
2.23	.48713	.03319	2.73	.49683	.00961
2.24	.48745	.03246	2.74	.49693	.00935
2.25	.48778	.03174	2.75	.49702	.00909
2.26	.48809	.03103	2.76	.49711	.00885
2.27	.48840	.03034	2.77	.49720	.00861
2.28	.48870	.02965	2.78	.49728	.00837
2.29	.48899	.02898	2.79	.49736	.00814
2.30	.48928	.02833	2.80	.49744	.00792
2.31	.48956	.02768	2.81	.49752	.00770
2.32	.48983	.02705	2.82	.49760	.00748
2.33	.49010	.02643	2.83	.49767	.00727
2.34	.49036	.02582	2.84	.49774	.00707
2.35	.49061	.02522	2.85	.49781	.00687
2.36	.49086	.02463	2.86	.49788	.00668
2.37	.49111	.02406	2.87	.49795	.00649
2.38	.49134	.02349	2.88	.49801	.00631
2.39	.49158	.02294	2.89	.49807	.00613
2.40	.49180	.02239	2.90	.49813	.00595
2.41	.49202	.02186	2.91	.49819	.00578
2.42	.49224	.02134	2.92	.49825	.00562
2.43	.49245	.02083	2.93	.49831	.00545
2.44	.49266	.02033	2.94	.49836	.00530
2.45	.49286	.01984	2.95	.49841	.00514
2.46	.49305	.01936	2.96	.49846	.00499
2.47	.49324	.01889	2.97	.49851	.00485
2.48	.49343	.01842	2.98	.49856	.00471
2.49	.49361	.01797	2.99	.49860	.00457

TABLE F — *Continued*

AREAS AND ORDINATES OF THE NORMAL CURVE OF ERROR IN TERMS OF ABSCISSA

x/σ	Area between maximum ordinate and ordinate at x/σ	Ordinate at x/σ (z)	x/σ	Area between maximum ordinate and ordinate at x/σ	Ordinate at x/σ (z)
3.00	.49865	.00443	3.50	.49977	.00087
3.01	.49869	.00430	3.51	.49978	.00084
3.02	.49874	.00417	3.52	.49978	.00081
3.03	.49878	.00405	3.53	.49979	.00079
3.04	.49882	.00393	3.54	.49980	.00076
3.05	.49886	.00381	3.55	.49981	.00073
3.06	.49889	.00370	3.56	.49981	.00071
3.07	.49893	.00358	3.57	.49982	.00068
3.08	.49897	.00348	3.58	.49983	.00066
3.09	.49900	.00337	3.59	.49983	.00063
3.10	.49903	.00327	3.60	.49984	.00061
3.11	.49906	.00317	3.61	.49985	.00059
3.12	.49910	.00307	3.62	.49985	.00057
3.13	.49913	.00298	3.63	.49986	.00055
3.14	.49916	.00288	3.64	.49986	.00053
3.15	.49918	.00279	3.65	.49987	.00051
3.16	.49921	.00271	3.66	.49987	.00049
3.17	.49924	.00262	3.67	.49988	.00047
3.18	.49926	.00254	3.68	.49988	.00046
3.19	.49929	.00246	3.69	.49989	.00044
3.20	.49931	.00238	3.70	.49989	.00042
3.21	.49934	.00231	3.71	.49990	.00041
3.22	.49936	.00224	3.72	.49990	.00039
3.23	.49938	.00216	3.73	.49990	.00038
3.24	.49940	.00210	3.74	.49991	.00037
3.25	.49942	.00203	3.75	.49991	.00035
3.26	.49944	.00196	3.76	.49992	.00034
3.27	.49946	.00190	3.77	.49992	.00033
3.28	.49948	.00184	3.78	.49992	.00031
3.29	.49950	.00178	3.79	.49992	.00030
3.30	.49952	.00172	3.80	.49993	.00029
3.31	.49953	.00167	3.81	.49993	.00028
3.32	.49955	.00161	3.82	.49993	.00027
3.33	.49957	.00156	3.83	.49994	.00026
3.34	.49958	.00151	3.84	.49994	.00025
3.35	.49960	.00146	3.85	.49994	.00024
3.36	.49961	.00141	3.86	.49994	.00023
3.37	.49962	.00136	3.87	.49995	.00022
3.38	.49964	.00132	3.88	.49995	.00021
3.39	.49965	.00127	3.89	.49995	.00021
3.40	.49966	.00123	3.90	.49995	.00020
3.41	.49968	.00119	3.91	.49995	.00019
3.42	.49969	.00115	3.92	.49996	.00018
3.43	.49970	.00111	3.93	.49996	.00018
3.44	.49971	.00107	3.94	.49996	.00017
3.45	.49972	.00104	3.95	.49996	.00016
3.46	.49973	.00100	3.96	.49996	.00016
3.47	.49974	.00097	3.97	.49996	.00015
3.48	.49975	.00094	3.98	.49997	.00014
3.49	.49976	.00090	3.99	.49997	.00014

TABLE F — *Continued*

AREAS AND ORDINATES OF THE NORMAL CURVE OF ERROR IN TERMS OF ABSCISSA

x/σ	Area between maximum ordinate and ordinate at x/σ	Ordinate at x/σ (z)	x/σ	Area between maximum ordinate and ordinate at x/σ	Ordinate at x/σ (z)
4.00	.49997	.00013	4.50	.50000	.00002
4.01	.49997	.00013	4.51	.50000	.00002
4.02	.49997	.00012	4.52	.50000	.00001
4.03	.49997	.00012	4.53	.50000	.00001
4.04	.49997	.00011	4.54	.50000	.00001
4.05	.49997	.00011	4.55	.50000	.00001
4.06	.49998	.00011	4.56	.50000	.00001
4.07	.49998	.00010	4.57	.50000	.00001
4.08	.49998	.00010	4.58	.50000	.00001
4.09	.49998	.00009	4.59	.50000	.00001
4.10	.49998	.00009	4.60	.50000	.00001
4.11	.49998	.00009	4.61	.50000	.00001
4.12	.49998	.00008	4.62	.50000	.00001
4.13	.49998	.00008	4.63	.50000	.00001
4.14	.49998	.00008	4.64	.50000	.00001
4.15	.49998	.00007	4.65	.50000	.00001
4.16	.49998	.00007	4.66	.50000	.00001
4.17	.49998	.00007	4.67	.50000	.00001
4.18	.49999	.00006	4.68	.50000	.00001
4.19	.49999	.00006	4.69	.50000	.00001
4.20	.49999	.00006	4.70	.50000	.00001
4.21	.49999	.00006	4.71	.50000	.00001
4.22	.49999	.00005	4.72	.50000	.00001
4.23	.49999	.00005	4.73	.50000	.00001
4.24	.49999	.00005	4.74	.50000	.00001
4.25	.49999	.00005	4.75	.50000	.00001
4.26	.49999	.00005	4.76	.50000	.00000
4.27	.49999	.00004	4.77	.50000	.00000
4.28	.49999	.00004	4.78	.50000	.00000
4.29	.49999	.00004	4.79	.50000	.00000
4.30	.49999	.00004	4.80	.50000	.00000
4.31	.49999	.00004	4.81	.50000	.00000
4.32	.49999	.00004	4.82	.50000	.00000
4.33	.49999	.00003	4.83	.50000	.00000
4.34	.49999	.00003	4.84	.50000	.00000
4.35	.49999	.00003	4.85	.50000	.00000
4.36	.49999	.00003	4.86	.50000	.00000
4.37	.49999	.00003	4.87	.50000	.00000
4.38	.49999	.00003	4.88	.50000	.00000
4.39	.49999	.00003	4.89	.50000	.00000
4.40	.49999	.00002	4.90	.50000	.00000
4.41	.49999	.00002	4.91	.50000	.00000
4.42	.50000	.00002	4.92	.50000	.00000
4.43	.50000	.00002	4.93	.50000	.00000
4.44	.50000	.00002	4.94	.50000	.00000
4.45	.50000	.00002	4.95	.50000	.00000
4.46	.50000	.00002	4.96	.50000	.00000
4.47	.50000	.00002	4.97	.50000	.00000
4.48	.50000	.00002	4.98	.50000	.00000
4.49	.50000	.00002	4.99	.50000	.00000

1.9
5198
12

This envelope contains samples of those types of co-ordinate paper that have been found most useful for statistical purposes.

To facilitate the work of students in Statistics, the authors have prepared several sets of laboratory material. These sets contain assortments of co-ordinate paper for use in connection with the series of problems suggested in Section III of this Manual. In addition to the co-ordinate paper, sets I-a, I-b and II contain twelve, and set III contains eight, heavy folders appropriately printed, in which the weekly laboratory reports should be submitted. This material is assembled in stout manila envelopes.

These sets may be obtained from the Codex Book Company, Inc., 461 Eighth Ave., New York City, at the prices noted below. Please order by number. Sets will be made up on order to correspond to any selection of problems.

Laboratory Set Number	Corresponding Set of Problems (See Section III)	Price per Set
I-a	I-a	$1.00
I-b	I-b	1.20
II	II	1.10
III	III	0.90